Computational strategies for flexible multibody systems

Tamer M Wasfy

Advanced Science and Automation Corp, Hampton VA
tamer@ascience.com

Ahmed K Noor

Center for Advanced Engineering Environments, Old Dominion University, Hampton VA;
a.k.noor@larc.nasa.gov

The status and some recent developments in computational modeling of flexible multibody systems are summarized. Discussion focuses on a number of aspects of flexible multibody dynamics including: modeling of the flexible components, constraint modeling, solution techniques, control strategies, coupled problems, design, and experimental studies. The characteristics of the three types of reference frames used in modeling flexible multibody systems, namely, floating frame, corotational frame, and inertial frame, are compared. Future directions of research are identified. These include new applications such as micro- and nano-mechanical systems; techniques and strategies for increasing the fidelity and computational efficiency of the models; and tools that can improve the design process of flexible multibody systems. This review article cites 877 references. [DOI: 10.1115/1.1590354]

1 INTRODUCTION

A flexible multibody system (FMS) is a group of interconnected rigid and deformable components, each of which may undergo large translational and rotational motions. The components may also come into contact with the surrounding environment or with one another. Typical connections between the components include: revolute, spherical, prismatic and planar joints, lead screws, gears, and cams. The components can be connected in closed-loop configurations (eg, linkages) and/or open-loop (or tree) configurations (eg, manipulators).

The term *flexible multibody dynamics* (FMD) refers to the computational strategies that are used for calculating the dynamic response (which includes time-histories of motion, deformation and stress) of FMS due to externally applied forces, constraints, and/or initial conditions. This type of simulation is referred to as *forward dynamics*. FMD also comprises *inverse dynamics*, which predicts the applied forces necessary to generate a desired motion response. FMD is important because it can be used in the analysis, design, and control of many practical systems such as: ground, air, and space transportation vehicles (such as bicycles, automobiles, trains, airplanes, and spacecraft); manufacturing machines; manipulators and robots; mechanisms; articulated earthbound structures (such as cranes and draw bridges); articulated space structures (such as satellites and space stations); and bio-dynamical systems (human body, animals, and insects). Motivated by these applications, FMD has been the focus of intensive research for the last thirty years. FMD is used in the design and control of FMS. In design, FMD can be used to calculate the system parameters (such as dimensions, configuration, and materials) that minimize the system cost while satisfying the design safety constraints (such as strength, rigidity, and static/dynamic stability). FMD is used in control applications for predicting the response of the multibody system to a given control action and for calculating the changes in control actions necessary to direct the system towards the desired response (inverse dynamics). FMD can be used in model-based control as an integral part of the controller as well as in controller design for optimizing the controller/FMS parameters.

In recent years, considerable effort has been devoted to modeling, design, and control of FMS. The number of publications on the subject has been steadily increasing. Lists and reviews of the many contributions on the subject are given in survey papers on FMD [1,2] and on the general area of multibody dynamics, including both rigid and flexible multibody systems [3–7]. Special survey papers have been published on a number of special aspects of FMD, including: dynamic analysis of flexible manipulators [8], dynamic analysis of elastic linkages [9–13], and dynamics of satellites with flexible appendages [14]. A number of books on FMD have been published [15–23]. In the last few years, there have been a number of conferences, symposia, and special sessions devoted to FMD [24]. Two archival journals are devoted to the subjects of rigid and flexible multibody dy-

Transmitted by Associate Editor V Birman

namics: ''Multibody System Dynamics'' published by Kluwer Academic Publishers, and ''Journal of Multibody Dynamics'' published by Ingenta Journals. There are a number of commercial codes for flexible multibody dynamics (eg, ADAMS from Mechanical Dynamics Inc, DADS from CADSI Inc, MECANO from Samtech, and SimPack from INTEC GmbH) as well as many research codes developed at universities and research institutions. A survey of multibody dynamics software up to 1990 with benchmarks was presented in Schiehlen [25]. There are two compelling motivations for developing FMD modeling techniques. The first motivation is that a number of current problems have not yet been solved to a satisfactory degree (see Section 9). The second motivation is that future multibody systems are likely to require more sophisticated models than has heretofore been provided. This is because practical FMD applications are likely to have more stringent requirements of economy, high performance, light weight, high speed/acceleration, and safety.

There is a need to broaden awareness among practicing engineers and researchers about the current status and recent developments in various aspects of FMD. The present paper attempts to fill this need by classifying and reviewing the FMD literature. Also, future directions for research that have high potential for improving the accuracy and computational efficiency of the predictive capabilities of the dynamics and failure of FMS are identified. Some of these objectives were addressed in the previous review papers. In the present paper, an attempt is made to provide a more comprehensive review of the literature. The following aspects of FMD are addressed in the present paper:

- Models of the flexible components
- Constraints models
- Solution techniques, including solution procedures and methods for enhancing the computational procedures and models
- Control strategies
- Coupled FMD problems
- Design of FMS
- Experimental studies

There are many common elements of FMD with structural dynamics, nonlinear finite element method and crashworthiness analysis. Some of the studies in these areas, which include techniques that are suitable for modeling FMS, are included in this review. The number of publications on the diverse aspects of FMD is very large. The cited references are selected for illustrating the ideas presented and are not necessarily the only significant contributions to the subject. The discussion in this paper is kept, for the most part, on a descriptive level and for all the mathematical details, the reader is referred to the cited literature.

2 MODELS OF FLEXIBLE COMPONENTS

2.1 Deformation reference frames

In multibody dynamics, an inertial frame serves as a global reference frame for describing the motion of the multibody system. In addition, intermediate reference frames that are attached to each flexible component and follow the average local rigid body motion (rotation and translation) are often used. The motion of the component relative to the intermediate frame is, approximately, due only to the deformation of the component. This simplifies the calculation of the internal forces because stress and strain measures that are not invariant under rigid body motion, such as the Cauchy stress tensor and the small strain tensor, can be used to calculate these forces with respect to the intermediate frame. These tensors result in a linear force displacement relation. Two main types of intermediate frames are used: floating and corotational frames. The floating frame follows an average rigid body motion of the entire flexible component or substructure. The corotational frame follows an average rigid body motion of an individual finite element within the flexible component. In many papers, intermediate frames are not used, instead the global inertial frame is directly used for measuring deformations. In this approach, the motion of an element consists of a combination of rigid body motion and deformation and the two types of motion are not separated. Nonlinear finite strain measures and corresponding energy conjugate stress measures, which are objective and invariant under rigid body motion, are used to calculate the internal forces with respect to the global inertial frame. A comparison between the major characteristics of the three types of frames, namely, floating, corotational, and inertial frames is given in Table 1. The references where the frames were first applied to FMS are given in Table 2.

The *floating frame approach* originated out of research on rigid multibody dynamics in the late 1960s. It was used for extending rigid multibody dynamics codes to FMS. This was done by superimposing small elastic deformations on the large rigid body motion obtained using the rigid multibody dynamics code. Initial applications of the floating frame approach included: spinning flexible beams (primarily for space structures applications), kineto-elastodynamics of mechanisms, and flexible manipulators (see Table 2). The floating frame approach was also used to extend modal analysis and experimental modal identification techniques to FMS [52,54,232,256,272]. This is performed by identifying the mode shapes and frequencies of each flexible component either numerically or experimentally. The first n modes (where n is determined by the physics of the problem and the by the required accuracy) are superposed on the rigid body motion of the component represented by the motion of the floating frame. In Table 3, a partial list of publications on the floating frame approach is organized according to the techniques used and developed and according to the type of application considered.

The *corotational frame approach* was initially developed as a part of the *natural mode method* proposed by Argyris *et al* [562]. In this approach, the motion of a finite element is divided into a rigid body motion and natural deformation modes. The approach was used for static modeling of structures undergoing large displacements and small deformations. Later, Belytschko and Hsieh [45] introduced element rigid convected frames or corotational frames, for the dy-

Table 1. Major characteristics of the three types of frames

	Floating Frame	Corotational Frame	Inertial Frame
Frame definition	A floating frame is defined for each flexible component. The floating frame of a component follows a mean rigid body motion of the component (see Fig. 1).	A corotational frame is defined for each element. The corotational frame of an element follows a mean rigid body motion of the element (see Fig. 2).	The global inertial reference frame is used as a reference frame for all motions (see Fig. 3).
Reference frame for:			
a) Deformation	**Floating frame (for each flexible component).**	**Corotational frame (for each finite element).**	**Global inertial reference frame.**
b) Internal forces	**Floating frame.** Note: In some implementations, the internal force components are transformed from the floating frame to the global inertial reference frame (eg, [26]).	**Corotational frame/Global inertial reference frame.** Note: The element internal force components are first calculated relative to the corotational frame, then they are transformed from the corotational frame to the global inertial frame using the corotational frame rotation matrix.	**Global inertial reference frame.** Note: The internal forces are calculated using finite strain measures which are invariant under rigid body motion.
c) Inertia forces	**Floating frame.** Note: In some implementations, the flexible motion inertia force components are first evaluated with respect to the global inertial reference frame and then are transformed to the floating frame (eg, [27,28]).	**Global inertial reference frame.** Notes: • In some implementations, the inertia force components are first evaluated relative to the corotational frame and then are transformed to the inertial frame (eg, [29–31]). • In spatial problems, for the rotational part of the equations of motion, the internal and inertia moments are often calculated relative to a moving material frame.	**Global inertial reference frame.** Note: In spatial problems, for the rotational part of the equations of motion, the internal and inertia moments are often calculated relative to a moving material frame.
Transformation to global inertial frame	Eq. (1).	Eq. (1).	No transformation is necessary.
Modeling Considerations			
a) Incorporation of flexibility effects.	The floating frame approach is the natural way to extend rigid multibody dynamics to flexible multibody systems.	The corotational frame transformation eliminates the element rigid body motion such that a linear deformation theory can be used for the element internal forces.	General finite strain measures that are invariant under superposed rigid body motion are used.
b) Magnitude of angular velocities	No restriction on angular velocities magnitudes. However, when linear modal reduction is used, the angular velocity should be low or constant because the stiffness of the body varies with the angular velocity due to the centrifugal stiffening effect [32].	No restriction on angular velocities magnitudes. In case of very small elastic deformations and large angular velocities, special care must be taken during the solution procedure (time step size, number of equilibrium iterations, etc) to avoid the situation where numerical errors from the rigid body motion are of the order of the elastic part of the response.	
c) Large deflections	• Moderate deflections can be modeled by using quadratic strain terms. However, large deflections cannot be modeled unless the body is sub-structured. • Without the assumption that the strains and deflections are small, the high-order terms of the flexible-rigid body inertial coupling terms cannot be neglected and the formulation becomes very complicated.	Can handle large deflections and large strains.	
d) Foreshortening	Foreshortening effect can be modeled by adding quadratic axial-bending strain coupling terms.	Naturally included.	
e) Centrifugal stiffening	Centrifugal stiffening can be modeled by adding the stress produced by the axial centripetal forces and including axial-bending strain coupling terms.	Naturally included.	
f) Mixing rigid and flexible bodies	Since the floating frame formulation is based on rigid multibody dynamics analysis methods, both rigid and flexible bodies can be present in the same model in any configuration with no difficulty.	Most implementations place some restrictions on the configuration of the rigid bodies, such as a closed-loop, must contain at least one flexible body.	

Table 1. *(continued)*

<table>
<tr><th></th><th>Floating Frame</th><th>Corotational Frame</th><th>Inertial Frame</th></tr>
<tr><td>Characteristics of the semi-discrete equations of motion</td><td>• The equations of motion are written such that the flexible body coordinates are referred to a floating frame and the rigid body coordinates are referred to the inertial frame.</td><td colspan="2">• The equations of motion are written with respect to the global inertial frame.
• In spatial problems with rotational DOFs, the rotational part of the equations of motion can be written with respect to a body attached nodal frame (material frame) [33–38] or with respect to the global inertial frame (spatial frame) [35,39].</td></tr>
<tr><td>a) Inertia forces</td><td>• The inertia forces involve nonlinear centrifugal, Coriolis, and tangential terms because the accelerations are measured with respect to a rotating frame (the floating frame).</td><td colspan="2">• The inertia forces are the product of the mass matrix and the vector of nodal accelerations with respect to the global inertial frame.
• In spatial problems with rotational DOFs, the rotational equations (the Euler equations) include quadratic angular velocity terms. (These terms vanish in planar problems.)</td></tr>
<tr><td></td><td>• The mass matrix has nonlinear flexible-rigid body motion coupling terms. The coupling terms are necessary for an accurate prediction of the dynamic response, when the magnitude of the flexible inertia forces is not negligible relative to that of the rigid body inertia forces.
• The solution procedure involves the inversion or the LU factorization of the time varying inertia matrices.</td><td colspan="2">• The translational part of the mass matrix is constant. Effects such as coupling between flexible and rigid body motion, centrifugal and coriolis acceleration are not present because the inertia forces are measured with respect to an inertial frame.</td></tr>
<tr><td>b) Internal (structural) forces</td><td>The internal forces are linear for small strains and slow rotational velocities. The linear part of the stiffness matrix is the same as that used in classical linear FEM. The nonlinear part of the stiffness matrix accounts for geometric nonlinearity and coupling between the axial and bending deformations (centrifugal stiffening effect).</td><td>For small strains, the internal forces are linear with respect to the corotational frame. The structural forces are transformed to the global frame using the nonlinear corotational frame transformation.</td><td>The internal forces are nonlinear even for small strains because they are expressed in terms of nonlinear finite strain and stress measures.</td></tr>
<tr><td>Constraints
a) Hinge joints</td><td>Hinge joints require the addition of algebraic constraint equations in the absolute coordinate formulation.</td><td colspan="2">Hinge joints (revolute joints in planar problems and spherical joints in spatial problems) do not need an extra algebraic equation and can be modeled by letting two bodies share a node.</td></tr>
<tr><td>b) General constraints</td><td>Constraints due to joints, prescribed motion and closed-loops are expressed in terms of algebraic equations. These equations must be solved simultaneously with the governing differential equations of motion. The development of general, stable, and efficient solution procedures for this system of differential-algebraic equations is still an active research area [40–42] (also see Section 4.1).</td><td colspan="2">Constraints due to joints and prescribed motion are expressed in terms of algebraic equations. If an implicit algorithm is used, then a system of differential-algebraic equations (DAEs) must be solved. If an explicit solution procedure is used, no special algorithm for solving DAEs is needed.</td></tr>
<tr><td>Applicability of linear modal reduction</td><td>• Can be applied.
• Can significantly reduce the computational time.
• Appropriate selection of the deformation components modes requires experience and judgment on the part of the analyst.
• For accuracy, linear modal reduction should be restricted to bodies undergoing slow rotation or uniform angular velocity.
• Nonlinear modal reduction [43,44] can be used for bodies undergoing fast non-uniform angular velocity in order to include the centrifugal stiffening effect. However, a modal reduction must be performed at each time step.</td><td>Not practical because the element vector of internal forces is nonlinear in nodal coordinates since it involves a rotation matrix.</td><td>Not practical because the element vector of internal forces is nonlinear in nodal coordinates since it involves a nonlinear finite strain measure.</td></tr>
<tr><td>Possibility of using modal identification experiments</td><td>The mode shapes and natural frequencies used in modal reduction can be obtained using experimental modal analysis techniques. Thus, there is a direct way to obtain the body flexibility information from experiments without numerical modeling.</td><td colspan="2">Experimentally identified modes cannot be directly used in the model. They can, however, be indirectly used to verify the accuracy of the predicted response and to tune the parameters of the model.</td></tr>
</table>

Table 1. *(continued)*

	Floating Frame	Corotational Frame	Inertial Frame
Most suitable applications	The floating frame formulation along with modal reduction and new recursive solution strategies (based on the relative coordinates formulation) offer the most efficient method for the simulation of flexible multibody systems undergoing small elastic deformations and slow rotational speeds (such as satellites and space structures).	The corotational and inertial frame formulations can handle flexible multibody systems undergoing large deflections and large high-speed rigid body motion. In addition, if used in conjunction with an explicit solution procedure, then high-speed wave propagation effects (for example, due to contact/impact) can be accurately modeled.	
Least suitable applications	Multibody problems, which involve large deflections.	For multibody problems involving small deformations and slow rotational speeds, the solution time is generally an order of magnitude greater than that of typical methods based on the floating frame approach with modal coordinates.	
First known application of the approach to FMS.	Adopted in the late 1960s to early 1970s to extend rigid multibody dynamics computer codes to flexible multibody systems.	Developed by Belytschko and Hsieh [45]. It was first applied to beam type FMS in Housner [46–48].	Used in nonlinear, large deformation FEM since the beginning of the 1970s. It was first applied to modeling beam type FMS in Simo and Vu-Quoc [49,50].

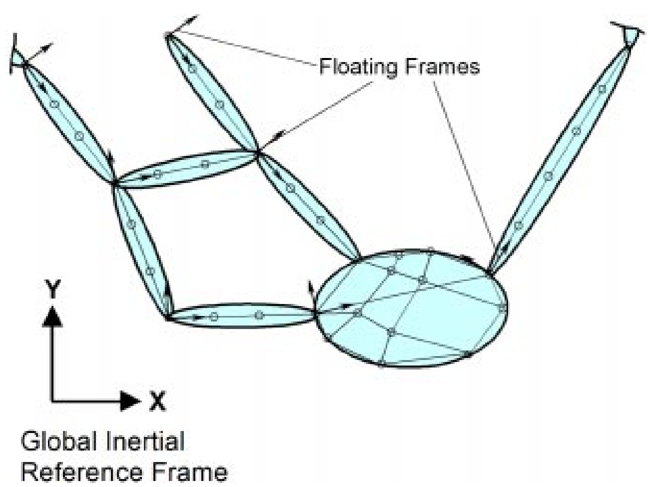

Fig. 1 Floating frame

namic modeling of planar continuum and beam type elements, using a total displacement explicit solution procedure. The approach was applied to spatial beams in Belytschko *et al* [33] and to curved beams in Belytschko and Glaum [452]. In Belytschko *et al* [468] and Belytschko *et al* [469], the approach was extended to dynamic modeling of shells using a velocity-based incremental solution procedure. Table 4 shows a partial list of publications which used corotational frames for developing computational models suitable for modeling FMS. The publications are organized according to the techniques used and developed and according to the type of application considered.

The *inertial frame approach* has its origins in the nonlinear finite element method and continuum mechanics principles. These techniques were applied to the dynamic analysis of continuum bodies undergoing large rotations and large deformations (including both large strains and large deflections) since the early 1970s [92,93]. In Table 5, publications where the inertial frame approach was used for developing computational models suitable for modeling FMS are classified.

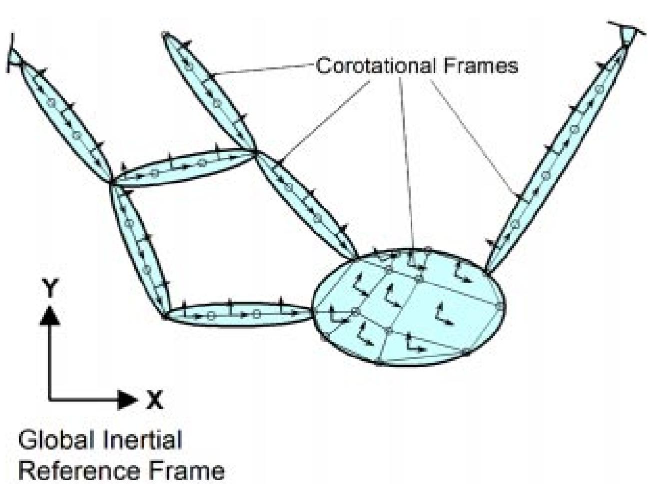

Fig. 2 Corotational frame

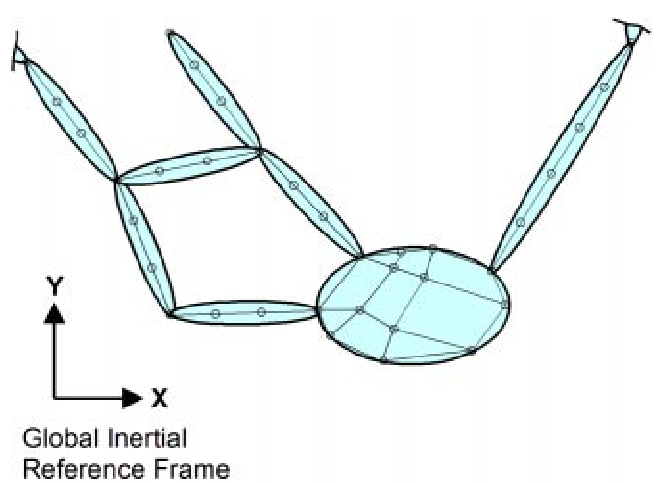

Fig. 3 Inertial frame

Table 2. Initial references for the application of the three types of frames to FMS

Floating Frame	Corotational Frame	Inertial Frame
Spinning beams: Meirovitch and Nelson [51], Likins [52,53,55], Likins *et al* [54], Grotte *et al* [56]. **Kineto-elastodynamics of mechanisms:** Winfrey [57–59], Jasinski *et al* [60,61], Sadler and Sandor [62], Erdman *et al* [9,63,64], Imam [65], Imam and Sandor [66], Viscomi and Ayre [67], Dubowsky and Maatuk [68], Dubowsky and Gardner [69,70], Bahgat and Willmert [71], Midha *et al* [72,74,75], Midha [73], Nath and Gosh [76], Huston [77], Huston and Passarello [78]. **Flexible manipulators:** Book [79,80].	**Nonlinear structural dynamics:** Belytschko and Hsieh [45], Belytschko *et al* [33], Argyris *et al* [81], Argyris [82], Belytschko and Hughes [83]. **Flexible space structures:** Housner [46], Housner *et al* [47]. **FMS planar beams:** Yang and Sadler [84], Wasfy [85,86], Elkaranshawy and Dokainish [31]. **FMS spatial beams:** Housner [46], Housner *et al* [47], Wu *et al* [87], Crisfield [88], Crisfield and Shi [89,90], Wasfy and Noor [91]. **FMS shells:** Wasfy and Noor [91].	**Nonlinear finite element method:** Oden [92], Bathe *et al* [93], Bathe and Bolourchi [94]. **Dynamics of planar flexible beams:** Simo and Vu-Quoc [50]. **Dynamics of spatial flexible beams:** Simo [95], Simo and Vu-Quoc [34,49,96,97], Iura and Atluri [48], Cardona and Geradin [35], Geradin and Cardona [98], Crespo Da Silva [99], Jonker [100].

2.2 Mathematical descriptions of the intermediate reference frames

The relation between the coordinates of a point in the global inertial frame A (x^A) and the coordinates of the same point in the intermediate body reference frame B (x^B) is given by:

$$x^A = x_o^{A/B} + R^{A/B} x^B \tag{1}$$

where $x_o^{A/B}$ are the coordinates of the origin of frame B in frame A, and $R^{A/B}$ is a rotation matrix describing the rotation from A to B. The methods used to define $x_o^{A/B}$ and $R^{A/B}$ for the floating and corotational frames are outlined subsequently.

2.2.1 Floating frame

The motion of the floating frame (position and orientation) is commonly referred to as the *reference motion* of the component. It is only an approximation of the rigid body motion of the component. Thus there are many ways to define this reference motion. Two formulations are commonly used, namely, fixed axis and moving axis formulations. In the fixed axis formulation, Cartesian and/or rotation coordinates of one, two, or three selected material points (usually the joints) on the flexible body are used to define the floating frame. Experience is needed for appropriate selection of body fixed axes that are consistent with the boundary conditions, because this choice affects the resulting vibrational modes. In the moving axis formulation, also called the body mean axis formulation, the floating frame follows a mean displacement of the flexible body and thus does not necessarily coincide with any specific material point. In this case, two definitions of the floating frame are used in practice: a) the floating frame is the frame relative to which the kinetic energy of the flexible motion with respect to an observer stationed at the frame is minimum (Tisserand frame) [109,122,123]; and b) the floating frame is the frame relative to which the sum of the squares of the displacements, with respect to an observer stationed at the frame, is minimum (Buckens frame) [122].

2.2.2 Corotational frame

The definition of the corotational frame depends on the type of elements used for modeling the flexible components. For two-node beam elements, the corotational frame is usually defined by the vector connecting the two nodes (eg, [45]). It can also be chosen as the mean beam axis (ie, the axis that minimizes the total deformation) [450]. For 3D beam elements, the remaining two axes are chosen as the cross-sectional axes [33,87,456]. In Park *et al* [479] and Cho *et al* [480] a relative nodal coordinate approach is used in which a tree representation of the FMS is constructed and beam element deformations are measured with respect to the adjacent nodal frame along the tree.

For shell and continuum elements, there are two methods to define the corotational frame. In the first method, only some of the nodes of the element are used to define the corotational frame. This type of definition was used for continuum elements in Belytschko and Hsieh [45] and for shells in Stolarski and Belytschko [455,456,468,470,471,563], Belytschko *et al* [468], Rankin and Brogan [455], Rankin and Nour-Omid [456], and Belytschko and Leviathan [470,471]. For example, in Belytschko *et al* [468] the normal Z-axis for a four node quadrilateral shell element is defined as the normal to the two diagonals of the element, the X-axis is perpendicular to the Z-axis and is aligned with the vector connecting nodes 1 and 2, and the Y-axis is perpendicular to the Z- and X-axes. Using some of the element nodes to define the corotational frame makes the internal forces dependent on the choice of the element local node numbering, which may introduce artificial asymmetries in the response [460,474,476]. In the second method, the origin and orientation of the corotational frame are defined as an average position and rotation of all the element nodes. For example, the origin of the corotational frame can be defined as the origin of the natural element coordinate system [85,91,460,464,474,476]. The orientation of the frame can be determined using one of the following techniques:

- Polar decomposition of the deformation gradient tensor at the origin of the natural element coordinate system [85,91,460,464,476]
- For shell elements, the Z-axis is normal to the surface of the element at the origin of the natural coordinate system. The angle between the X-axis and the first element natural axis is equal to the angle between the Y-axis and the second element natural direction [564]
- A least-square minimization procedure to find the orienta-

Table 3. Classification of a partial list of publications on the floating frame approach

Definition of the floating frame	**Coordinates of selected points (body fixed axis)**	Most references, *eg* Winfrey [57,58], Sadler and Sandor [62,101,102], Song and Haug [103], Sunada and Dubowsky [104,105], Shabana and Wehage [106], Singh *et al* [107], Turcic and Midha [108], Agrawal and Shabana [109], Changizi and Shabana [110], Ider and Amirouche [111–113], Chang and Shabana [114], Modi *et al* [115], Shabana and Hwang [116], Hwang and Shabana [117], Pereira and Nikravesh [118].
	Mean rigid body motion (moving axis)	Likins [52], Milne [119], McDonough [120], Fraejis de Veubeke [121], Canavin and Likins [122], Cavin and Dusto [123], Agrawal and Shabana [109,124], Koppens *et al* [125].
Rigid body coordinates	**Absolute coordinates (Augmented formulation)**	Most references, *eg* Song and Haug [103], Yoo and Haug [126,127], Shabana and Wehage [106], Agrawal and Shabana [109], Bakr and Shabana [128,129].
	Relative (or joint) coordinates (recursive formulation)	**Open-loop rigid multibody systems** Chace [130], Wittenburg [131], Roberson [132]. **Open-loop FMS (tree configuration)** Hughes [133], Hughes and Sincarsin [134], Book [135], Singh *et al* [107], Usoro *et al* [136], Benati and Morro [137], Changizi and Shabana [110], Kim and Haug [138], Han and Zhao [139], Shabana [140,141], Shabana *et al* [142], Shareef and Amirouche [143], Amirouche and Xie [144], Surdilovic and Vukobratovic [145], Znamenacek and Valasek [146]. **Closed-loop FMS** Kim and Haug [147], Ider and Amirouche [111,112], Keat [148], Nagarajan and Turcic [149], Lai *et al* [150], Ider [151], Pereira and Proenca [152], Nikravesh and Ambrosio [153], Jain and Rodriguez [154], Hwang [155], Hwang and Shabana [117,156], Shabana and Hwang [116], Amirouche and Xie [144], Verlinden *et al* [157], Tsuchia and Takeya [158], Pereira and Nikravesh [118], Fisette *et al* [159], Pradhan *et al* [160], Choi *et al* [161], Nagata *et al* [162].
3D finite rotation Description	**Euler angles**	Ider and Amirouche [111,112], Amirouche [17], Modi *et al* [115], Du and Ling [163].
	Euler parameters	Nikravesh *et al* [164], Agrawal and Shabana [109], Geradin *et al* [165], Haug *et al* [166]. Yoo and Haug [126], Wu and Haug [167], Wu *et al* [168], Chang and Shabana [114,169,170], Ambrosio and Goncalaves [171].
	Two unit vectors	Vukasovic *et al* [172].
	Rotation vector	Metaxas and Koh [173].
	Three vectors (rotation tensor)	Garcia de Jalon *et al* [174], Garcia de Jalon and Avello [175], Friberg [176], Bayo *et al* [177].
Inertial coupling between rigid body motion and flexible body motion (tangential, Coriolis and centrifugal inertia forces).		**Special formulations (initial research)** Viscomi and Ayre [67], Sadler and Sandor [102], Sadler [178], Chu and Pan [179], Cavin and Dusto [123]. **FMS formulations** Song and Haug [103], Haug *et al* [166], Nath and Gosh [76], Shabana and Wehage [106,180], Turcic and Midha [108,181], Shabana [182], Bakr and Shabana [128,129], Shabana [141], Hsu and Shabana [183], El-Absy and Shabana [184], Shabana [21], Yigit *et al* [185], Liou and Erdman [26], Ider and Amirouche [111,112], Dado and Soni [186], Naganathan and Soni [187], Nagarajan and Turcic [149], Silverberg and Park [188], Liu and Liu [189], Huang and Wang [190], Jablokow *et al* [191], Shabana *et al* [142], Shabana and Hwang [116], Lieh [192], Hu and Ulsoy [193], Fang and Liou [194], Damaren and Sharf [195], Xianmin *et al* [196], Shigang *et al* [197], Al-Bedoor and Khulief [198], Langlois and Anderson [199].
Centrifugal stiffening		**Single Rotating Body** Likins *et al* [54], Likins [55], Vigneron [200], Levinson and Kane [201], Kaza and Kvaternik [202], Cleghorn *et al* [203], Wright *et al* [204], Kane *et al* [205], Kammer and Schlack [206], Ryan [207], Trindade and Sampaio [208]. **FMS** Ider and Amirouche [111,112], Liou and Erdman [26], Peterson [209], Banerjee and Dickens [210], Banerjee and Lemak [211], Banerjee [212], Wallrapp *et al* [213], Wallrapp [214], Boutaghou and Erdman [215], Huang and Wang [190], Liu and Liu [189], Ryu *et al* [43], Hu and Ulsoy [193], Sharf [216,217], Yoo *et al* [218], Du and Ling [163], Tadikonda and Chang [219], Damaren and Sharf [195], Pascal [220]. **Studies on the effect of centrifugal stiffening on the response of FMS** Wallrapp and Schwertassek [221], Padilla and Von Flotow [222], Khulief [32], Zhang *et al* [223], Zhang and Huston [224], Ryu *et al* [44].
Geometric nonlinearity or moderate deflections.		**Beams** Bakr and Shabana [128,129], Spanos and Laskin [225], Hu and Ulsoy [193], Mayo *et al* [226], Mayo and Dominguez [227], Du *et al* [228], Sharf [216,217,229], Shabana [21].
Axial foreshortening		Meirovitch [230], Kane *et al* [205], Ryan [207], Hu and Ulsoy [193], Mayo *et al* [226], Mayo and Dominguez [227], Sharf [217], Ruzicka and Hodges [231].

Table 3. *(continued)*.

Model reduction		**Normal modes superposition** Likins [52,53], Winfrey [59], Imam *et al* [232], Likins *et al* [54], Sunada and Dubowsky [104], Hablani [233,235,236],Amirouche and Huston [234], Bakr and Shabana [128], Yoo and Haug [126,127,237], Tsuchiya *et al* [238], Chadhan and Agrawal [239], Nikravesh [240], Jonker [241], Ramakrishnan *et al* [242], Padilla and Von Flotow [222], Wang [243], Jablokow *et al* [191], Ramakrishnan [244], Yao *et al* [245], Wu and Mani [246], Verlinden *et al* [157], Hsieh and Shaw [247], Korayem *et al* [248], Hu *et al* [249], Tadikonda [250], Nakanishi *et al* [251], Lee [252], Cuadrado *et al* [253], Subrahmanyan and Seshu [254], Fisette *et al* [159], Shabana [21], Znamenacek and Valasek [146], Pan and Haug [255], Craig [256]. **Effect of Centrifugal stiffening on the reduced modes** Likins *et al* [54], Likins [55], Vigneron [200], Laurenson [257], Hoa [258], Wright *et al* [204], Banerjee and Dickens [210], Banerjee and Lemak [211], Khulief [32], Ryu *et al* [43,44], Kobayashi *et al* [259], Mbono Samba and Pascal [260]. **Selection of deformation modes** Kim and Haug [261], Friberg [262], Spanos and Tsuha [263], Tadikonda and Schubele [264], Gofron and Shabana [265], Shabana [266], Shi *et al* [267], Carlbom [268]. **Use of experimental Modes** Shabana [269]. **Effect of rigid-flexible motion coupling on the reduced modes** Shabana [270], Shabana and Wehage [106,180], Agrawal and Shabana [109], Hu and Skelton [271], El-Absy and Shabana [184], Friberg [262], Hablani [236], Jablokow *et al* [191], Cuadrado *et al* [253]. **Craig-Bampton modes** Craig and Bampton [272], Craig [256], Ryu *et al* [273,274], Cardona [275]. **Singular Perturbation model reduction** Siciliano and Book [276], Jonker and Aarts [277]. **Substructuring (Superelements)** Subbiah *et al* [278], Shabana [279], Shabana and Chang [280], Wu and Haug [281], Cardona and Geradin [282], Liu and Liew [283], Lim *et al* [284], Mordfin [285], Haenle *et al* [286], Liew *et al* [287], Cardona [275]. **Super-element for rigid multibody systems** Agrawal and Chung [288], Agrawal and Kumar [289]. **Effect of Geometric Nonlinearity** Wu and Haug [167,281], Wu and Mani [246].
Element types	**Beam**	**Planar Euler Beam** Bakr and Shabana [128], Liou and Erdman [26], Boutaghou and Erdman [215], Padilla and Von Flotow [222], Langlois and Anderson [199]. **Spatial Euler-Beam** Sharan *et al* [290], Richard and Tennich [291], Ghazavi *et al* [292], Sharf [216,217,229], Du and Ling [163], Du and Liew [293]. **Planar Timoshenko beam** Naganathan and Soni [187,294], Ider and Amirouche [111–113], Boutaghou and Erdman [215], Smaili [295], Hu and Ulsoy [193], Meek and Liu [296], Xianmin *et al* [196]. **Spatial Timoshenko beam** Christensen and Lee [297], Naganathan and Soni [187,294], Bakr and Shabana [129], Gordaninejad *et al* [298], Huang and Wang [190], Fisette *et al* [159], Oral and Ider [299], Shabana [21]. **Curved Beam** Bartolone and Shabana [300], Gau and Shabana [301], Chen and Shabana [302]. **Twisted Beams** Kane *et al* [205]. **Arbitrary Cross-Sections** Kane *et al* [205].
	Plates and shells	**Kirchhoff-Love Theory** Banerjee and Kane [303], Chang *et al* [304], Chang and Shabana [114,169], Boutaghou *et al* [305], Kremer *et al* [306,307], Madenci and Barut [308]. **Initially Curved plates:** Chen and Shabana [302,309].
	Continuum	Turcic and Midha [108,181], Turcic *et al* [310], Shareef and Amirouche [143], Jiang *et al* [311], Ryu *et al* [312], Fang and Liou [194].
Discretization	**Finite elements**	**Most references.**
	Boundary element	Kerdjoudj and Amirouche [313].
	Finite difference	Feliu *et al* [314].
	Analytical	Meirovitch and Nelson [51], Neubauer *et al* [315], Jasinski *et al* [60,61], Viscomi and Ayre [67], Sadler [178], Thompson and Barr [316], Badlani and Kleinhenz [317], Low [318,319], Boutaghou *et al* [320], Xu and Lowen [321,322]. **Symbolic Manipulation:** Cetinkunt and Book [323], Fisette *et al* [159,324], Korayem *et al* [248], Botz and Hagedorn [325,326], Piedboeuf [327], Melzer [328], Oliviers *et al* [329], Shi and McPhee [330,331], Shi *et al* [267,332].
Variable configuration	**Variable kinematic structure**	Khulief and Shabana [333,334], Ider and Amirouche [113], Chang and Shabana [170], Fang and Liou [194].
	Variable mass	McPhee and Dubey [335], Hwang and Shabana [336], Kovecses *et al* [337].

Table 3. ***(continued).***

Joints	**Prismatic**	Buffinton and Kane [338], Pan [339], Pan *et al* [340,341], Hwang and Haug [342], Gordaninejad *et al* [343], Buffinton [344], Al-Bedoor and Khulief [198,345], Verlinden *et al* [157], Fang and Liou [194], Theodore and Ghosal [346].
	Gears	Cardona [347].
	Cams	Bagci and Kurnool [348].
Material models	**Linear isotropic**	**Most references.**
	Composite	**Solid beam cross section:** Shabana [349]. **Box cross section:** Ider and Oral [350], Oral and Ider [299]. Thompson *et al* [351], Thompson and Sung [352], Sung *et al* [353], Shabana [349], Azhdari *et al* [354], Chalhoub *et al* [355], Gordaninejad *et al* [343], Kremer *et al* [306,307], Madenci and Barut [308], Du *et al* [228], Ghazavi *et al* [292], Gordaninejad and Vaidyaraman [356], Ider and Oral [350], Oral and Ider [299].
	Nonlinear	**Plastic materials for crash analysis:** Ambrosio [357], Ambrosio and Nikravesh [27]. **Inelastic materials:** Amirouche and Xie [358], Pan and Haug [255]. **Creeping materials:** Xie and Amirouche [359].
Coupling with other effects	**Control** **Piezo-electric actuators**	Gofron and Shabana [360,361], Gordaninejad and Vaidyaraman [356]. Rose and Sachau [362].
	Thermal	Shabana [363], Sung and Thompson [364], Modi *et al* [115].
	Aeroelasticity	Du *et al* [365,366].
Equations of Motion	**Lagrange's equations**	Dubowsky and Gardner [69], Midha *et al* [72,367], Midha *et al* [74], Blejwas [368], Cleghorn *et al* [203], Turcic and Midha [108,181], Book [135], Bakr and Shabana [128], Changizi and Shabana [110], Pan *et al* [341], Bricout *et al* [369], Meirovitch and Kwak [370], Smaili [295], Pereira and Proenca [152], Modi *et al* [115], Huang and Wang [190], Meek and Liu [296], Fattah *et al* [371], Metaxas and Koh [173], Pereira *et al* [372], Pradhan *et al* [160].
	Hamilton's principle	Cavin and Dusto [123], Serna [373], Fung [374].
	Kane's equations	Ider and Amirouche [111,112], Banerjee and Dickens [210], Ider [151], Han and Zhao [139], Amirouche and Xie [144], Zhang *et al* [223], Zhang and Huston [224], Langlois and Anderson [199].
	Newton-Euler equations	Naganathan and Soni [187,294], Huang and Lee [375], Shabana [140,376], Hwang [155], Hwang and Shabana [117,156], Shabana *et al* [142], Richard and Tennich [291], Verlinden *et al* [157], Hu and Ulsoy [193], Ambrosio [377], Choi *et al* [161].
	Principle of Virtual Work	Liu and Liu [189], Lieh [192], Shi and McPhee [330].
Mass matrix	**Consistent**	**Most references.**
	Lumped	Lai and Dopker [378], Han and Zhao [139], Shabana [376], Jain and Rodriguez [154], Pan and Haug [379], Ambrosio and Ravn [28], Ambrosio and Goncalaves [171].
Solution	**Iterative implicit**	**Most references.**
Procedure	**Explicit**	Metaxas and Koh [173].
Applications	**Mechanisms (Closed-Loops)**	**Review papers:** Lowen and Jandrasits [10], Lowen and Chassapis [12], Thompson and Sung [13]. **Planar:** Winfrey [57,58], Sadler and Sandor [62,101,102], Sadler [178,380], Jasinski *et al* [60,61], Erdman *et al* [63], Viscomi and Ayre [67], Chu and Pan [179], Thompson and Barr [316], Bahgat and Willmert [71], Midha *et al* [72,74,75], Badlani and Kleinhenz [317], Song and Haug [103], Nath and Gosh [76,381], Cleghorn *et al* [203], Blejwas [368], Bagci and Abounassif [382], Badlani and Midha [383], Turcic and Midha [108,181], Turcic *et al* [310], Thompson and Sung [352], Tadjbakhsh and Younis [384], Sung and Thompson [364], Liou and Erdman [26], Cardona and Geradin [282], Banerjee [212], Jablokow *et al* [191], Liou and Peng [385], Hsieh and Shaw [247], Verlinden *et al* [157], Fallahi *et al* [386], Chassapis and Lowen [387], Sriram and Mruthyunjaya [388], Sriram [389], Farhang and Midha [390], Yang and Park [391], Xianmin *et al* [196], Fung [374], Subrahmanyan and Seshu [254]. **Spatial:** Sunada and Dubowsky [104], Shabana and Wehage [106,392], Hwang and Shabana [117].
	Space Structures	**Review paper:** Modi [14] Meirovitch and Nelson [51], Ashley [393], Likins [52,53], Likins *et al* [54], Grotte *et al* [56], Kulla [394], Canavin and Likins [122], Ho [395], Bodley *et al* [396], Lips and Modi [397], Kane and Levinson [398,399], Kane *et al* [400], Bainum and Kumar [401], Diarra and Bainum [402], Hablani [233,235,236], Laskin *et al* [403], Modi and Ibrahim [404], Ibrahim and Modi [405], Ho and Herber [406], Wang and Wei [407], Meirovitch and Quinn [408], Meirovitch and Quinn [409], Tsuchiya *et al* [238], Man and Sirlin [410], Hanagud and Sarkar [411], Silverberg and Park [188], Meirovitch and Kwak [370], Spanos and Tsuha [263], Modi *et al* [115], Kakad [412], Wu and Chen [413], Wu *et al* [414], Tadikonda *et al* [415], Tadikonda [416], Pradhan *et al* [160], Nagata *et al* [162].

Table 3. ***(continued).***

Manipulators (tree)	**Review paper:** Gaultier and Cleghorn [8]. **Chains** Hughes [133], Hughes and Sincarsin [134], Wang [243]. **Manipulators (open-loops)** Book [79,135], Sunada and Dubowsky [105], Judd and Falkenburg [417], Subbiah *et al* [278], Bricout *et al* [369], Chang and Hamilton [418], Chang [419], Chedmail *et al* [420], Geradin *et al* [421], Singh *et al* [107], Serna [373], Gordaninejad *et al* [343], Han and Zhao [139], Pascal [422], Sharan *et al* [142,290], Smaili [295], Huang and Wang [190], Yao *et al* [245], Hu and Ulsoy [193], Fattah *et al* [371], Meek and Liu [296], Du and Ling [163], Du and Liew [293], Liew *et al* [287], Surdilovic and Vukobratovic [145], Oral and Ider [299], Theodore and Ghosal [346], Shigang *et al* [197], Kovecses *et al* [337].
Rotorcraft	Du *et al* [365,366,423], Bertogalli *et al* [424], Ruzicka and Hodges [231].
Vehicle dynamics	**Review paper:** Kortum [425]. Pereira and Proenca [152], Richard and Tennich [291], Schwartz [426], Kading and Yen [427], Sharp [428], Nakanishi and Shabana [429], Tadikonda [250], Nakanishi *et al* [251], Nakanishi and Isogai [430], Pereira *et al* [372], Campanelli *et al* [431], Choi *et al* [161], Lee *et al* [432], Assanis *et al* [433], Carlbom [268], Ambrosio and Goncalaves [171].
Human body dynamics	Amirouche and Ider [434], Amirouche *et al* [435].
Crash-worthiness	Nikravesh *et al* [436], Ambrosio [377].

tion that minimizes the sum of the squares of the difference between the orientations of the element sides and the corotational frame orientation [474])
- Finding the orientation that makes the rotation at the origin of the corotational frame zero [478]

The last two approaches are difficult to extend for elements with mid-side nodes and for 3D solid elements [476].

In most FMS applications, the element deformations are small and, therefore, one corotational frame per element is sufficient. If the deformation within the element is large, such as in large-strain problems, then one corotational frame per element may not be sufficient to approximate the rigid body motion of the entire element. In this case, more than one corotational frame per element that follows the average local element rigid body motion are needed. However, using more than one corotational element per frame is contrary to the main advantage of the corotational frame approach, which is simplicity and computational efficiency of the element internal forces.

2.3 Semi-discrete equations of motion

The semi-discrete equations of motion of a FMS involve two types of equations: the dynamic equations of equilibrium and constraint equations. The dynamic equilibrium equations can be written as:

$$F_I = F_N + F_E + F_R \tag{2}$$

where F_I, F_N, F_E, and F_R are the vectors of inertia, internal, external, and constraint forces, respectively. Constraint equations express the relations between the various components of the system. They have the following form:

$$\Phi(q,\dot{q},t) \geqslant 0 \tag{3}$$

where Φ is the vector of algebraic constraint equations, q is the vector of generalized system coordinates, t is the time, and a superposed dot indicates a time derivative. In the floating frame approach, Eq. (2) is usually written such that the flexible body coordinates are referred to a floating frame and the rigid body coordinates (which define the motion of the floating frames) are referred to the inertial frame. In the corotational and inertial frame approaches, Eq. (2) is usually written for the entire multibody system with respect to the global inertial reference frame. The inertial and internal force vectors in Eq. (2) for the floating, corotational, and inertial frame approaches can be written in the following form:

Floating frame:

For a flexible component:

$$\begin{aligned} q &= \begin{Bmatrix} q_R \\ q_F \end{Bmatrix} \\ F_I &= M(q)\ddot{q} + F_c \\ F_N &= K q_F \end{aligned} \tag{4}$$

Corotational frame:

For an individual finite element:

$$\begin{aligned} q &= \begin{Bmatrix} x \\ \theta \end{Bmatrix} \\ F_I &= \begin{Bmatrix} M\ddot{x} \\ J\ddot{\theta} + \dot{\theta}\times J\dot{\theta} \end{Bmatrix} \\ F_N &= RKq_F \end{aligned} \tag{5}$$

Inertial frame:

For an individual finite element:

$$\begin{aligned} q &= \begin{Bmatrix} x \\ \theta \end{Bmatrix} \\ F_I &= \begin{Bmatrix} M\ddot{x} \\ J\ddot{\theta} + \dot{\theta}\times J\dot{\theta} \end{Bmatrix} \\ F_{N_{t+\Delta t}} &= F_{N_t} + K_{T_t}\Delta q \end{aligned} \tag{6}$$

Table 4. Classification of a partial list of publications on the corotational frame approach

Element types	**Beams**	**Planar Euler beam** Belytschko and Hsieh [45], Hsiao and Jang [29,437], Hsiao *et al* [438], Yang and Sadler [84], Rice and Ting [439], Tsang [440], Tsang and Arabyan [441], Iura [442], Mitsugi [443], Hsiao and Yang [444], Elkaranshawy and Dokainish [31], Wasfy [86], Galvanetto and Crisfield [445], Shabana [21,446], Shabana and Schwertassek [447], Banerjee and Nagarajan [448], Behdinan *et al* [449], Takahashi and Shimizu [450], Berzeri *et al* [451]. **Planar Curved Euler beam** Belytschko and Glaum [452], Hsiao and Yang [444]. **Planar Timoshenko beam** Iura and Iwakuma [30], Iura and Atluri [453]. **Spatial Euler beam** Belytschko *et al* [33], Argyris *et al* [81,454], Bathe and Bolourchi [94], Housner [46], Housner *et al* [47], Rankin and Brogan [455], Rankin and Nour-Omid [456], Wu *et al* [87,457], Crisfield [88,458], Hsiao [459], Wasfy [85,460], Wasfy and Noor [91]. **Spatial Timoshenko beams** Quadrelli and Atluri [461,462], Crisfield *et al* [38], Devloo *et al* [463].
	Shells	Rankin and Brogan [455], Rankin and Nour-Omid [456]. **Kirchhoff-Love model** Peng and Crisfield [464], Wasfy and Noor [91], Shabana and Christensen [465], Meek and Wang [466]. **Mindlin model** Belytschko and Tsay [467], Belytschko *et al* [468,469], Belytschko and Leviathan [470,471], Bergan and Nygard [472,473], Nygard and Bergan [474].
	Continuum	Belytschko and Hsieh [45], Argyris *et al* [81], Belytschko and Hughes [83], Flanagan and Taylor [475], Wasfy [85,460]. Wasfy and Noor [91], Crisfield and Moita [476], Moita and Crisfield [477].
Definition of the corotational frame	**Defined with respect to the position of selected element nodes**	**Beams** All references. **Shells and Continuum** Belytschko *et al* [468], Belytschko and Leviathan [470,471], Rankin and Brogan [455], Rankin and Nour-Omid [456], Meek and Wang [466].
	Defined with respect to a mean rigid body motion of the element	**Shells** Nygard and Bergan [474], Wasfy and Noor [91]. **Continuum** Flanagan and Taylor [475], Jetteur and Cescotto [478], Wasfy [85,460], Wasfy and Noor [91], Crisfield and Moita [476].
	Defined with respect to the previous element (relative nodal coordinates)	Park *et al* [479], Cho *et al* [480]
Beam and shell 3D rotation parameters	**Euler angles**	**Beams:** Bathe and Bolourchi [94].
	Incremental rotation vector	**Beams** Quadrelli and Atluri [461,462]. **Shells** Bergan and Nygard [472,473], Nygard and Bergan [474], Belytschko *et al* [468,469], Belytschko and Leviathan [470,471].
	Rotation vector	**Beams** Crisfield [88], Crisfield *et al* [38], Devloo *et al* [463]. **Shells** Argyris *et al* [81,454], Argyris [82], Rankin and Brogan [455], Rankin and Nour-Omid [456].
	Two unit vectors	**Beams:** Belytschko *et al* [33].
Deformation reference	**Total Lagrangian**	Belytschko and Hsieh [45], Belytschko and Glaum [452], Hughes and Winget [481], Flanagan and Taylor [475], Hsiao and Jang [29,437], Yang and Sadler [84], Crisfield [88], Rice and Ting [439], Tsang [440], Tsang and Arabyan [441], Wasfy [85,86,460], Wasfy and Noor [91], Hsiao [459], Hsiao *et al* [438], Hsiao and Yang [444], Crisfield and Shi [89], Crisfield and Moita [476], Elkaranshawy and Dokainish [31], Iura and Atluri [453], Quadrelli and Atluri [461,462], Crisfield *et al* [38], Shabana [21,446], Shabana and Christensen [465], Behdinan *et al* [449], Takahashi and Shimizu [450].
	Updated Lagrangian	Bathe and Bolourchi [94], Belytschko *et al* [468,469], Belytschko and Leviathan [470,471], Jetteur and Cescotto [478], Quadrelli and Atluri [461,462], Meek and Wang [466].
Mass matrix	**Lumped**	Belytschko and Hsieh [45], Belytschko and Glaum [452], Rice and Ting [439], Wasfy [85,86,460], Wasfy and Noor [91], Iura and Atluri [453].
	Consistent	Hsiao and Jang [29,437], Hsiao *et al* [438], Yang and Sadler [84], Tsang [440], Wu *et al* [87], Tsang and Arabyan [441], Hsiao and Yang [444], Elkaranshawy and Dokainish [31], Crisfield *et al* [38], Shabana [446], Shabana and Christensen [465], Devloo *et al* [463].
DOFs	**Rotations and displacements**	**Most references**, eg, Belytschko and Hsieh [45], Belytschko *et al* [33], Bathe and Bolourchi [94], Rankin and Brogan [455], Rankin and Nour-Omid [456], Yang and Sadler [84], Crisfield *et al* [38], Devloo *et al* [463].
	Cartesian Displacements	Wasfy [85,86,460], Wasfy and Noor [91], Banerjee [482], Banerjee and Nagarajan [448].
	Slopes and displacements	Shabana [21,446,483], Shabana and Christensen [465], Shabana and Schwertassek [447], Takahashi and Shimizu [450], Berzeri *et al* [451].

Table 4. ***(continued)***

Solution procedure	**Implicit**	**Semi-implicit with Newton iterations** Housner [46], Housner *et al* [47,484], Yang and Sadler [84], Hsiao and Jang [29], Hsiao *et al* [438], Hsiao and Yang [444], Elkaranshawy and Dokainish [31], Banerjee and Nagarajan [448], Behdinan *et al* [449], Shabana [21], Devloo *et al* [463], Cho *et al* [480]. **Energy conserving:** Crisfield and Shi [89,90], Galvanetto and Crisfield [445], Crisfield *et al* [38].
	Explicit	Belytschko and Hsieh [45], Belytschko and Kennedy [485], Hallquist [486], Flanagan and Taylor [475], Taylor and Flanagan [487], Rice and Ting [439], Wasfy [85,86,460], Wasfy and Noor [91], Iura and Atluri [453]. **Multi-time Step:** Belytschko *et al* [488], Belytschko and Lu [489].
Material models	**Linear isotropic**	**Most references.**
	Composite Materials	
	Hyper-elastic materials	Crisfield and Moita [476].
	Elastic-plastic	Flanagan and Taylor [475].
Governing equations of motion	**Lagrange equations**	Yang and Sadler [84], Elkaranshawy and Dokainish [31], Tsang and Arabyan [441], Devloo *et al* [463].
	Hamilton's principle	Iura and Atluri [453].
	Virtual work/ D'Alembert Principle	Housner [46], Housner *et al* [47], Wu *et al* [87], Crisfield [88], Wasfy [85,460], Wasfy and Noor [91].
Applications	**Nonlinear structural dynamics**	Belytschko and Hsieh [45], Belytschko *et al* [33], Rice and Ting [439].
	Crashworthiness	Belytschko *et al* [468], Belytschko [490], Belytschko and Leviathan [470,471].
	Space structures	Housner [46], Housner *et al* [47], Wu *et al* [87], Wasfy and Noor [91], Banerjee and Nagarajan [448].
	General FMS (mechanisms and manipulators)	Yang and Sadler [84], Wasfy [85,86,460], Elkaranshawy and Dokainish [31], Wasfy and Noor [91], Shabana [446], Shabana and Christensen [465].

where q_R is the vector of rigid body translations and rotations with respect to the global inertial reference frame, q_F is the vector of flexible coordinates (displacements and slopes) with respect to the intermediate frame, M is the mass matrix, F_c is the vector of coriolis and centrifugal inertia forces, K is a constant stiffness matrix, x is the vector of element nodal coordinates with respect to the global inertial frame, θ is the vector of element nodal rotations with respect to a material frame or the global inertial frame, J is the matrix of rotational inertia, K_T is a linearized tangent stiffness matrix, t is the running time, Δt is the time increment, and Δq is the vector of translation and rotation increments.

In Eq. (4), the expression of the inertia forces for the floating frame involves nonlinear Coriolis, centrifugal, and tangential inertia forces that are the result of using the noninertial floating frame as the reference frame. The Coriolis and centrifugal terms are included in F_C, while the tangential acceleration term makes the mass matrix a function of the flexible coordinates. The nonlinear inertia terms couple the rigid body and flexible body motions. The internal forces, on the other hand, are linear provided that the deformations with respect to the intermediate frame and the angular velocities are small (see Subsection 2.9).

Equations (5) and (6) follow from the Newton-Euler equations of motion. In these equations, the expression of the translational part of the inertia forces for the corotational and inertial frame is just mass times acceleration because these forces are referred to an inertial frame. The expression of the rotational part of the inertia forces includes a quadratic angular velocity term ($\dot{\theta}\times J\dot{\theta}$). This term is only present in problems involving spatial rotations, and vanishes for planar problems. The rotational part of the equations of motion can be referred to either a moving material frame, or to the global inertial reference frame. In the first case J is constant, while in the second case J is constant for planar problems and is time varying in spatial problems. The expression of the internal forces is nonlinear because it involves either a rotation matrix (which is a function of q) in the case of the corotational frame, or nonlinear finite strain and stress measures in the case of the inertial frame. For the corotational frame, if the strains are small and the material is linearly elastic, the linearity of the force-displacement relation is maintained at the element level before multiplying by R (see Eq. (5)). In other words, the nonlinearity due to large rotations appears only in the transformation of the internal forces from the corotational to the inertial frame.

In the majority of implementations of the floating frame, the inertia and internal forces are written in a similar form as in Eq. (4), which means that Eq. (2) is written for a flexible component with respect to the floating frame of the component. This choice allows the use of modal reduction methods, which can greatly reduce the computational cost. In a few

Table 5. Classification of a partial list of publications on the inertial frame approach

Element types	**Beams**	**Planar Euler beam** Gontier and Vollmer [491], Gontier and Li [492], Meijaard [493], Meijaard and Schwab [494], Shabana [21], Berzeri and Shabana [495], Berzeri *et al* [451]. **Planar Timoshenko Beams** Simo and Vu-Quoc [50], Ibrahimbegovic and Frey [496], Stander and Stein [497]. **Planar Curved Timoshenko beams** Ibrahimbegovic and Frey [496], Ibrahimbegovic [498]. **Spatial Euler-Beam** Rosen *et al* [499]. **Spatial Timoshenko Beams** Simo [95], Simo and Vu-Quoc [34,49,96,97], Vu-Quoc and Deng [500], Cardona and Geradin [35], Geradin and Cardona [98], Iura and Atluri [48,501], Crespo Da Silva [99], Avello *et al* [39], Park *et al* [502], Downer *et al* [36], Downer and Park [503], Borri and Bottasso [504], Bauchau *et al* [505], Ibrahimbegovic and Frey [506], Ibrahimbegovic *et al* [507], Ibrahimbegovic and Al Mikdad [37], Bauchau and Hodges [508]. **Bifurcation and instability in Spatial Timoshenko Beams** Cardona and Huespe [509,510]. **Spatial curved Timoshenko Beams (Reissner beam theory)** Ibrahimbegovic [498], Ibrahimbegovic and Mamouri [511], Ibrahimbegovic *et al* [512], Borri *et al* [513]. **Continuum mechanics principles** Wasfy [514].
	Plates and Shells	**Kirchhoff-Love model** Rao *et al* [515]. **Mindlin-Reissner model** Simo and Fox [516], Simo *et al* [517], Simo and Tarnow [518], Vu-Quoc *et al* [519], Ibrahimbegovic [520,522], Ibrahimbegovic and Frey [506,521], Boisse *et al* [523], Bauchau *et al* [524]. **Degenerate shell theory** Hughes and Liu [525], Mikkola and Shabana [526]. **Continuum mechanics principles** Parisch [527], Wasfy and Noor [528], Wasfy [514].
	Continuum	Oden [92], Bathe *et al* [93], Laursen and Simo [529], Bathe [530], Kozar and Ibrahimbegovic [531], Ibrahimbegovic *et al* [512], Goicolea and Orden [532], Orden and Goicolea [533], Wasfy [514].
Rigid body, beam, and shell 3D rotation description	**Euler-Parameters**	Spring [534], Park *et al* [502], Downer *et al* [36].
	Rotational pseudo-vector (Semi-tangential rotations)	Argyris [82], Park *et al* [502], Downer *et al* [36].
	Incremental rotation vector	Ibrahimbegovic [522,535], Bauchau *et al* [524], Ibrahimbegovic and Mamouri [511], Borri *et al* [513].
	Conformal rotation vector (quaternion)	Geradin and Cardona [98], Bauchau *et al* [505], Lim and Taylor [536].
	Rotation vector	Simo [95], Simo and Vu-Quoc [34,49,97], Simo and Fox [516], Cardona and Geradin [35], Geradin and Cardona [98], Borri and Bottasso [504], Ibrahimbegovic and Frey [521], Kozar and Ibrahimbegovic [531], Ibrahimbegovic *et al* [507], Ibrahimbegovic and Al Mikdad [37].
	Two unit vectors	Avello *et al* [39].
	Rotation tensor	Simo and Vu-Quoc [34,49,97], Avello *et al* [39], Ibrahimbegovic and Frey [521], Ibrahimbegovic [498], Ibrahimbegovic *et al* [507], Ibrahimbegovic and Mamouri [511], Bauchau *et al* [505], Boisse *et al* [523].
DOFs	**Rotations and displacements**	**Most references.**
	Cartesian Displacements	Parisch [527], Goicolea and Orden [532], Orden and Goicolea [533], Wasfy and Noor [528], Wasfy [514].
	Slopes and displacements	Berzeri and Shabana [495], Berzeri *et al* [451], Mikkola and Shabana [526].
Beam shape Functions	**Polynomial**	**Most references.**
	Bezier functions	Gontier and Vollmer [491].
	Helicoid	Borri and Bottasso [504].
	Load-dependent modes	Meijaard and Schwab [494].
	Eigen modes	Meijaard and Schwab [494].
Mass matrix	**Lumped**	Park *et al* [502], Downer *et al* [36], Wasfy and Noor [528], Wasfy [514].
	Consistent	**Most references.**
Deformation Reference	**Total Lagrangian**	Bathe *et al* [93], Nagarajan and Sharifi [537], Simo and Vu- Quoc [34,49,50], Cardona and Geradin [35], Ibrahimbegovic and Frey [506,521], Kozar and Ibrahimbegovic [531], Ibrahimbegovic and Al Mikdad [37], Boisse *et al* [523], Wasfy and Noor [528], Wasfy [514], Campanelli *et al* [538], Goicolea and Orden [532], Orden and Goicolea [533], Berzeri and Shabana [495], Mikkola and Shabana [526].
	Updated Lagrangian	Bathe *et al* [93], Cardona and Geradin [35], Boisse *et al* [523].

Table 5. *(continued).*

Governing Equations of Motion	**D'Alembert Principle**	Wasfy and Noor [528], Wasfy [514], Berzeri and Shabana [495], Mikkola and Shabana [526].
	Hamilton's principle	Bauchau *et al* [505], Bauchau *et al* [524].
	Lagrange equations	Hac [539,540], Hac and Osinski [541].
Material Models	**Linear elastic**	**Most references.**
	Composite materials	Vu-Quoc *et al* [519,542,543], Vu-Quoc and Deng [500], Bauchau and Hodges [508], Ghiringhelli *et al* [544].
Solution procedure	**Implicit**	Simo and Vu-Quoc [34,49,50], Cardona and Geradin [35], Ibrahimbegovic and Al Mikdad [37], Goicolea and Orden [532], Orden and Goicolea [533], Berzeri and Shabana [495], Mikkola and Shabana [526], Nagarajan and Sharifi [537], Geradin *et al* [545].
	Explicit	Park *et al* [502,546], Downer *et al* [36], Wasfy and Noor [528], Wasfy [514].
	Hybrid	**Implicit-Explicit multi-time step:** Vu-Quoc and Olsson [547–549].
Applications	**Non-linear structural dynamics**	Oden [92], Bathe *et al* [93], Bathe [530], Parisch [527].
	Vehicle dynamics	Vu-Quoc and Olsson [547–550]. **Belt-Drives:** Leamy and Wasfy [551–553].
	Flexible space structures	Vu-Quoc and Simo [554], Wasfy [514]. **Mechanical deployment:** Wasfy and Noor [528]. **Attitude control:** Wasfy and Noor [528].
	Tethered satellites	**Tether deployment:** Leamy *et al* [555]. **Vibration control:** Dignath and Schiehlen [556].
	Rotorcraft	Ghiringhelli *et al* [544], Bauchau *et al* [557].
	General FMS (mechanisms and manipulators)	Van der Werff and Jonker [558], Jonker [100,559], Simo and Vu-Quoc [49,50], Cardona and Geradin [35], Park *et al* [502,546], Downer *et al* [36], Bauchau *et al* [505,560], Hac [539,540], Wasfy [514].
	Axially moving media	Vu-Quoc and Li [561].

implementations of the floating frame, Eq. (2) is written with respect to the global inertial frame (see Table 1). These implementations do not allow the use of modal reduction. In addition, only small deflections are allowed within a body unless nonlinear strain measures are used.

In the majority of implementations of the corotational frame, the inertia and internal forces are written in a similar form as in Eq. (5), which means that Eq. (2) is written with respect to the global inertial frame. This allows the use of a simple expression for the translational part of the inertia forces. Also, the internal forces are linear with respect to the corotational frame (provided the strains are small and the constitutive relations are linear). The internal forces are first evaluated with respect to the corotational frame and are then transferred to the global inertial frame using the rotation matrix of the corotational frame. In a few implementations of the corotational frame, Eq. (2) is first written with respect to the element corotational frame and then it is transformed to the global inertial frame (see Table 1). The disadvantage of this approach is that the translational mass matrix includes nonlinear terms [30].

2.4 Deformation of the flexible components

The kinematic relations for different types of structural members can be classified into different groups according to the spatial extent of the members. Beam models are used for 1D members; plate and shell models are used for 2D members; and continuum models are used for 3D members. These models are used in conjunction with the floating, corotational, and inertial frames in FMD applications. Tables 3-5 provide a partial list of publications where these models are used in FMS. Brief descriptions of these models is presented subsequently, along with the issues related to the use of each model in conjunction the choice of reference frame.

2.4.1 Beam elements

Beam elements are used in the majority of FMD publications due to the fact that many flexible components are long and slender. Two categories of beam models are used: Euler-Bernoulli beam model and Timoshenko beam model. In the Euler-Bernoulli model, the transverse shear deformation is neglected and the beam cross sections are assumed to remain plane, rigid, and normal to the beam neutral axis after deformation. The Euler-Bernoulli models provide a good approximation for beams with cross-sectional dimensions less than one tenth the beam length. The rotations of the cross section of a beam can be expressed in terms of the displacement derivatives with respect to the axial coordinate of the beam. Thus, the rotation of the beam cross section and the displacement are not independent. The governing partial differential equation relating the transverse structural forces to the deformation involves a fourth-order derivative with respect to the spatial coordinate. Therefore, if a single-field displacement model is used, shape-functions with C^1 continuity are used for the transverse displacements (cubic polynomial for two-node beams). For the axial displacements, only C^0 continuity is needed for the shape functions (linear polynomial for two-node beams). Using different shape functions for the transverse and axial displacements can be easily implemented in floating and corotational frame formulations. In the inertial frame formulations, since all displacements are measured with respect to the inertial frame and there is no distinction between transverse and axial displacements, the same interpolations are used for all displacements with respect to the inertial frame. Thus, inertial frame formulations do not use Euler-Bernoulli beam theory. Also note that in Euler-Bernoulli beams rotary inertia (inertia due to the rotation of the cross section) is often neglected because the theory is suitable only for thin beams, for which rotary inertia is small.

The Timoshenko beam model accounts for shear deformation. The rotations of the beam cross section and the displacement are independent and the beam cross sections remain plane after bending, but not necessarily normal to the beam neutral axis. Timoshenko beam theory is a good approximation for thick beams with length of more than three times the cross-sectional dimensions. Shape functions with C^0 continuity are usually used for the displacement and rotation components. All inertial frame beam implementations reported in the literature are based on Timoshenko beam theory. As mentioned above, this is because all motions are referred to the inertial frame; therefore interpolation functions should not distinguish between transverse and axial displacements. Thus, all displacement and rotational DOFs are interpolated independently using the same interpolation functions, which are linear functions for two-node beam elements [34,35,50,453,498,507]. Timoshenko beams are also extensively used in conjunction with both floating and corotational frame formulations (see Tables 3 and 4). Finally, note that all Timoshenko beam implementations include the rotary inertia because Timoshenko beams are suitable for thick beams for which rotary inertia is important.

A difficulty of Timoshenko beam theory is that it leads to shear locking for thin beams. Techniques to remedy shear locking include: reduced and selective reduced integration of the internal forces [35,496], enhanced interpolations [496], and the assumed strain method. Some techniques to avoid shear locking, such as reduced integration may give rise to spurious oscillation modes. Iura and Atluri [453] used the exact solution for linear static Timoshenko beams to derive the stiffness operator with respect to the corotational frame and demonstrated that this approach eliminates shear locking.

Euler-Bernoulli and Timoshenko beams have only one axial dimension. Those elements can support bending in one of the following ways:

- Using rotational DOFs at the element nodes. Most references use this technique. Many types of rotation parameters are used (eg, Euler angles, Euler parameters, and rotation vectors.) Tables 3-5 list the references which use each type of rotation parameters. Also, a discussion of the rotation parameters is given in Subsection 2.6.
- Using global slope DOFs at the element nodes [446,483]
- Using the torsional spring formulation where the inter-element slopes are measured using the local nodal displacements [5,15,86,91,448,460]

Many types of kinematic couplings between tangential (axial) and transverse displacements are present in beams. These couplings arise due to the geometry of the beam. Typical kinematic couplings that have been considered are: beam curvature, arbitrary cross sections, and twisted (or warped) beams (coupling of torsion and bending). Tables 3-5 provide a partial list of the references where kinematic couplings are considered in conjunction with the floating, corotational, and inertial frames.

Most references use polynomial shape functions for the beam elements such as linear or third order polynomials. In some references new types of interpolations are suggested such as: Bezier functions [491] and helicoid [504].

2.4.2 Shell and solid elements

Three types of shell models are used: Kirchhoff-Love models, Reissner-Mindlin models, and degenerate shell models. In addition, shells can be modeled using solid elements that are based on continuum mechanics principles.

Kirchhoff-Love models for shells are the 2D counterparts of Euler-Bernoulli models for beams. They assume that normals to the shell reference surface remain straight and normal after deformation and are inextensional. These models are only valid for thin shells. Transverse displacements and slopes over the shell must be continuous when Kirchhoff-Love models are used. For four-node shell elements, a bicubic interpolation for transverse displacements is needed, while in-plane displacements are interpolated using a bilinear interpolation. Using different interpolations for the transverse and axial displacements is allowed only in a floating or corotational frame formulation.

Reissner-Mindlin type models incorporate shear deformation and are the 2D counterparts of Timoshenko models for beams. The rotations and transverse displacements are independent [468] and normals to the shell reference surface remain straight and inextensional but not necessarily normal. The degenerate shell models are based on 3D continuum mechanics with a collapsed thickness coordinate [525,565]. Solid elements do not collapse the thickness coordinate and thus do not have to use rotational DOFs. Inertial frame shell implementations are based on either the Reissner-Mindlin or continuum mechanics principles. This is due to the fact that since all motions are referred to the inertial frame, interpolation functions should not distinguish between transverse and in-plane displacements, and all displacement and rotational DOFs are interpolated independently using the same interpolation functions such as bi-linear functions for four-node shell elements [468,523].

Shell and solid elements are used in many types of loading conditions such as bending, tension, compression, shear, torsion, and coupled combinations of the previous loadings. Many elements proposed in the literature give accurate results under certain types of loading and poor results under other types of loading. In addition, many elements perform poorly if the element shape is distorted [566]. In order to test the overall accuracy and robustness of an element, standard tests problems have been proposed which include many combinations of loadings and element distortions [567,568]. Ideally, a shell or solid element should pass all those tests. The main reason for poor shell and solid elements performance is locking. Many types of locking can occur [530,563,569–571], including:

- *Shear locking* is caused by the overestimation of shear strains when the element is undergoing pure bending due to low order interpolation.
- *Membrane locking* is caused by the overestimation of the membrane strains for curved elements when the element is undergoing pure bending.
- *Trapezoidal locking* is related to membrane locking and is caused by the fact that when the element is distorted (trapezoidal shape) the membrane forces are not aligned with the element edges. Thus they cause a moment that resists bending.
- *Thickness locking* is also related to membrane locking and is caused by the activation of transverse normal strains due to the Poisson ratio terms when the element is undergoing pure bending.
- *Volumetric locking* occurs when a nearly incompressible material (Poisson ratio close to 0.5) is used.

Locking can occur in the plane of the element for shell elements. In addition, Reissner-Mindlin theory and the degenerate shell theory lead to shear locking in the transverse direction. Four techniques are available to eliminate or alleviate locking:

- Reduced integration methods
- Assumed field methods
- Natural-modes elements
- Higher-order elements

Reduced integration methods. Reduced integration serves two functions: reducing the computational cost of the element and remedying locking [467,563,572]). Unfortunately, if reduced integration is used, then the element bending modes (hourglass modes) are not modeled and, accordingly, they become spurious zero energy modes. Adding artificial strains, which are orthogonal to all linear fields (thus they are not activated by constant straining or by rigid body motion), can stabilize these modes [467,469,573]. In early implementations, ad hoc user-input hourglass control parameters were used to calculate the associated artificial stress. The global response was found to be sensitive in some cases to these parameters [574]. The ad hoc parameters were later eliminated [470,471,478,574] by using the Hu-Washizu variational principle to determine the magnitude of the stabilization parameters. Stabilized reduced integration elements cannot model bending with only one element through the thickness because they do not have a physically correct bending mode. Even two to three layers of elements may not provide accurate results. In Harn and Belytschko [575], an adaptive procedure is devised in which the number of quadrature points for the normal stresses is changed depending on the deformation state of the element.

Assumed field methods. The main reason for locking in shell and solid elements is the use of the classical isoparametric formulation where the deformation field is assumed to be given by the element interpolation functions. For low order linear elements, this deformation field cannot accurately capture the combined bending and shear deformations. Assumed field methods include: the method of incompatible modes [476,521,531,576,577], assumed natural strain [527,578], enhanced-strain [523,577], and assumed stress [579]. In the assumed field methods, a strain, stress, or deformation gradient field is added to the strain field obtained using the element isoparametric shape functions so as to allow the occurrence of pure bending deformation modes with vanishing shear. Some of those techniques introduce extra variables that can be eliminated using static condensation. Those techniques, in most cases, are used with the fully integrated element.

Natural modes elements. Some researchers proposed abandoning the isoparametric formulation in favor of a *natural deformation modes* formulation [81]. In this formulation, the element natural deformation modes are used as a basis for constructing the element stiffness matrix. For example, the TRIC triangular shell element [580–582] is divided into three beams with each beam possessing four natural deformation modes (extension, shear, symmetric bending, and asymmetric bending). In a triangular element that uses three truss sub-elements to model the membrane behavior and three torsional spring sub-elements to model the bending behavior was presented. In Wasfy and Noor [528] and Wasfy [514], an eight-node solid brick element that consists of twelve truss sub-elements and six surface shear sub-elements with appropriate stiffness and damping values for modeling the brick natural deformation modes (three membrane, six bending, three asymmetric bending, three shear, and three warping modes), was developed. Natural modes elements can be designed to avoid locking while accurately modeling the element deformation modes.

Higher-order elements. Another way to reduce locking is to use second and third order isoparametric Lagrangian elements. Third order elements have a bending mode that is nearly shear free and therefore suffer negligible shear locking. Lee and Bathe [566] showed that the 16-node planar rectangular Lagrangian element has negligible shear and membrane locking if its sides are straight and the mid-side nodes are evenly spaced. Higher order elements have been seldom used in FMS applications because:

- They suffer membrane locking when they are curved [571].
- They are computationally expensive.
- They are more complex and involve more DOFs.
- Mesh generation is more difficult.
- Mid-side and corner nodes are not equivalent. This makes it difficult to connect them to other elements and joints. Also, it complicates the formulation and modeling of their inertia characteristics and their use in contact/impact problems.

- Their accuracy, stability, and locking behavior are sensitive to the location of the mid-side nodes.

2.5 Treatment of large rotations

A major characteristic of FMS is that the flexible components undergo large rigid body rotations. The treatment of large rotations in the floating, corotational, and inertial frame approaches is discussed subsequently.

2.5.1 Floating frame

In the floating frame approach, large rotations are handled at the component level using the component's floating frame. The deformation of the flexible components is described by small displacement and slope DOFs that are defined relative to the floating frame. The fact that the component is moving and rotating introduces nonlinear inertia coupling, tangential, centrifugal, and Coriolis terms in the inertia forces, and a centrifugal stiffening effect in the internal forces. These terms are discussed in Subsection 2.8. The position and orientation of each floating frame (or flexible component), with respect to the global inertial reference frame, can be determined using three position coordinates and a minimum of three orientation coordinates. The position coordinates define the origin of the floating frame and the orientation coordinates define the rotation matrix (R) of the floating frame (Eq. (1)). Commonly used orientation angles are the three Euler angles. However, it is known that the use of three parameters to define the spatial orientation of a body leads to singularities at certain orientations. Thus, researchers prefer to use non-minimal spatial orientation descriptions such as Euler parameters, two unit vectors, rotation vector, or rotation tensor (see Table 3). The various types of spatial orientation descriptions were first used in rigid multibody dynamics and then ported to FMD. Note that in planar problems there is no problem with rotation parameterization because the orientation of the floating frame is easily defined using only one angle.

2.5.2 Corotational and inertial frame

In the inertial and corotational frame formulations, the final expression of the internal force vector of a finite element involves a rotation or deformation gradient matrix which:

- Defines the local rigid body rotation
- Transforms the DOFs relative to the inertial frame to local DOFs
- Transforms the local internal forces back to the inertial frame

When modeling beams and shells, rotational DOFs are often used. The types of nodal rotation parameters used in conjunction with the corotational frame and inertial frames are listed in Tables 4 and 5, respectively. Many researches use more than one type of rotation parameters. For example, in Park *et al* [502] and Downer et al. [36], the rotational pseudo vector is used for calculating the internal forces and Euler parameters are used for the time integration. Reviews of the different types of rotation parameters and the relations between them are given in Argyris [82], Spring [534], Atluri and Cazzani [535,583], Ibrahimbegovic [535], Betsch *et al* [584], and Borri *et al* [585]. Spatial finite rotations can be uniquely represented using a second-order orthogonal rotation tensor Ψ. The six orthogonality conditions ($\Psi\Psi^{T}=I$) can be used to reduce the representation to a minimum of three. There are a number of difficulties associated with rotational DOFs:

- Using three parameters (eg, Euler angles) or four parameters (eg, rotation vector) lead to singularities at certain positions. For example, for rotation magnitudes greater than π, the rotation vector at a node is not unique [35,37]. This singularity can be removed using a correction routine for rotations greater than π. Alternatively, the incremental rotation vector [35] can be used. Incremental rotation vectors are additive, can be transformed as vectors, and are free of singularities [35,474].
- The relation between the various rotation parameters and the generalized physical moments and the moments of inertia involve complicated trigonometric functions.
- In spatial problems with rotational DOFs, the rotational part of the equations of motion can be written with respect to the global inertial frame (spatial frame) [35,39] or a body attached nodal frame (material frame) [33–38]. Referring the rotational equations to the inertial frame in spatial problems leads to a moment of inertia tensor which varies with time, thus requiring it to be computed every time step. On the other hand, if the rotational equations are written with respect to a material frame, then the moment of inertia tensor with respect to that frame is constant.
- Interpolation of different types of rotational DOFs (such as Euler angles, Euler parameters, rotation vector, etc) is not equivalent.
- Interpolation of incremental and total rotation measures spoils the objectivity of the strain measure with respect to rigid body rotation [586]. In addition, interpolation of incremental rotations, especially in the inertial frame approach, leads to accumulation of rotation errors in a path dependent way [538,586].
- Drilling rotational DOFs were used in shell elements [472–474,587], membrane elements [506,520,521, 588,589], as well as solid elements [531]. This makes the element compatible with beam elements. However, it was shown in Ibrahimbegovic and Frey [521] that the introduction of drilling rotational DOFs can amplify the shear locking effect. The accuracy of the element was recovered by using the method of incompatible modes to remedy the shear locking [521,531].

Recently, in Shabana [446,483,590,591], an *absolute nodal coordinates formulation* was developed, in which global slope DOFs are used instead of rotational DOFs. This leads to an isoparametric formulation with a constant mass matrix. The formulation was first used with a corotational type frame for planar beams. Then it was used with the global inertial frame as the only reference frame in Berzeri and Shabana [495] and Berzeri *et al* [451]. The application of this formulation to spatial problems requires the use of 12 DOFs (three translational DOFs and nine slope DOFs) per node [465,526] as opposed to only six DOFs per node (three translational

DOFs and three rotational DOFs) for elements that use traditional rotational DOFs. More research is being conducted to develop elements that use this formulation and assess their accuracy, convergence, robustness, and computational efficiency.

In order to circumvent the difficulties associated with rotational and slope DOFs, some researches use only Cartesian nodal coordinates to model beams and shells. In this case, the equations of motion are written with respect to the global inertial frame and the mass matrix is constant. The treatment of large rotations, in this case, is straightforward (requiring a rotation or deformation gradient matrix). The kinematic condition necessary for modeling beams and shells is that the transverse displacements and slopes between elements are continuous (this condition may be satisfied only in a global sense). This condition can be satisfied at element interfaces without using rotational DOFs by using the vectors connecting the nodes to define the inter-element slopes or by using solid elements. Three-node torsional spring beam formulations [5,15,85,86,91,448,482,592] achieve slope continuity between elements by using the direction of the vector connecting two successive nodes as the direction of the tangent to the beam at the midpoint between the two nodes. This technique was also used to develop a triangular three-node shell element in Argyris *et al* [593] and an eight-node shell element in Wasfy and Noor [91]. The latter element exhibits negligible locking because it has the correct bending modes. However, the element has the same difficulties of other high-order elements outlined at the end of Section 2.4.

Recently, many researchers developed displacement-based solid elements, based on continuum mechanics principles, that can be used to model beams and shells:

- Hexahedral eight-node element [527,571,594]
- Pentagonal six-node element [595]
- Hexahedral 18-node element with two layers of nodes each having nine nodes (thus the thickness direction is linearly interpolated) [571,596]. This high-order element exhibits the difficulties outlined at the end of Section 2.4.

All the above elements used the assumed natural strain or stress methods to remedy locking. Unfortunately, those elements have only been tested in static and quasi-static large deformation problems, but have not yet been tested in dynamic problems. In Wasfy and Noor [528] and Wasfy [514] the natural-modes eight-noded brick element based on the inertial reference frame was designed to accurately model the element deformation modes while avoiding locking and spurious modes. It was shown in Wasfy [514] that the element accurately solved standard benchmark dynamic shell and beam problems. The element was also used to simulate the deployment process of a large articulated space structure over 180 sec. The model consisted of beams, shells, revolute joints, prismatic joints, linear actuators, rotary actuators, and PD tracking controllers.

2.6 Reference configuration

Two reference configuration choices are used in practice: total Lagrangian (TL) and updated Lagrangian (UL). In the TL formulation, the reference configuration is the unstressed configuration (or the initial configuration at time 0). In the UL formulation, the reference configuration is the configuration at the previous time step. The UL and TL formulations can be used with the floating, corotational, or inertial frame approaches (see Tables 3–5).

In UL formulations the stress-strain relation is more naturally expressed in rate form relating a stress rate tensor to an energy conjugate strain-rate tensor. Jaumann stress rate [530] is often used in inertial frame formulations and Cauchy stress rate [468,597] is often used in corotational frame formulations. UL formulations are used in conjunction with corotational [468–471] and inertial [94,444,475,481] frame formulations in large strain applications such as crash-worthiness, metal forming, and nonlinear structural dynamics. Those applications often involve plastic material behavior. UL formulations are most suited for systems which involve large strains and plastic material behavior because the constitutive stress-strain relations used in these applications, such as visco-plastic material models, are usually expressed in terms of strain and stress rates Bathe [468,530]. In UL formulations, because the stress state at each time step depends on the computed stress state at the previous time step, numerical errors such as iteration errors, time integration errors, and round-offs can accumulate from one time step to the next causing the response to drift in time [439,449]. This drift is much more critical in FMD applications because they involve much larger rigid body rotations (which usually involves many revolutions) and much longer simulation times relative to metal forming and crash-worthiness applications. The response drift is more critical in implicit methods than in explicit methods [468] because the chosen time step is usually much larger than the smallest time step of the system, thus resulting in larger time integration errors. Also, the response drift is more critical for inertial frame formulations than corotational frame formulations because the latter eliminate the rigid body rotation before the UL stress update. Park *et al* [502] and Downer *et al* [36] developed a corotational UL formulation along with an explicit solution procedure to model spatial Timoshenko beams. Meek and Wang [466] developed a corotational UL formulation along with an implicit solution procedure for modeling shells.

Many inertial and corotational frame formulations use an UL formulation for rotations in which rotations are described as increments with respect to the configuration at the previous time step (eg, [35,474]). This formulation is very convenient because incremental rotations are vector quantities and, therefore, are additive and free of singularities. However, Jelenic and Crisfield [586] showed that, similar to the UL stress update, this can lead to accumulation of rotation errors in a path dependent way.

TL formulations do not suffer from the response drift problem during stress updates because the strain is always referred to a fixed known configuration. Most floating frame formulations use a TL formulation because displacements relative the floating frame are relatively small and, thus, there is no advantage in using an UL formulation. Also, most corotational frame formulations (see Table 4) and inertial frame formulations (see Table 5), which are developed specifically for FMS, use a TL formulation.

2.7 Discretization techniques

In the majority of FMD literature on floating, corotational, and inertial frame approaches, the flexible components are discretized using the finite element method. Other discretization techniques have been used in conjunction with the floating frame approach. These are:

- Normal mode technique (see Modal Reduction in Subsection 2.8.4)
- Finite differences [101,178]
- Boundary element method [313]
- Element-free Galerkin method (EFGM) [598]
- Analytical modeling [11,60,66,67,315–316]. In analytical modeling techniques, generally only one link of the multibody system is assumed to be elastic while the others are rigid.

2.8 Special modeling techniques used in conjunction with the floating frame

Since the equations of motion (Eq. (3)) for the floating frame are written with respect to the floating frame, which is a non-inertial frame, special modeling techniques are needed to handle the nonlinear inertia forces. In addition, other special modeling techniques which are used in conjunction with the floating frame approach include: the description of rigid body motion in terms of absolute or relative coordinates, treatment of geometric nonlinearities, and modal reduction methods. Table 3 lists the references where these techniques were developed.

2.8.1 Absolute and relative coordinates

An important classification of rigid body coordinates of the floating frame is whether absolute or relative coordinates are used. In the absolute coordinates formulation, the coordinates of each body are referred to the global inertial reference frame. Joints and motion constraints couple and constrain the rigid body coordinates of the bodies (such that they are no longer independent). This method is also called the *augmented formulation* because the resulting equations of motion involve sparse matrices and a non-minimal number of DOFs that include six spatial degrees of freedom for each body, Lagrange multipliers associated with the constraints between the bodies, and elastic coordinates of each body. The formulation simplifies the introduction of general constraint and forcing functions for both open and closed-loop FMS.

In the relative coordinates formulation, the coordinates of a body in a chain of bodies are expressed in terms of the coordinates of the previous body in the chain and the DOFs of the joint connecting the two bodies. Thus, for open-loop systems, the generalized coordinates are independent and their number is minimal. This formulation is also called the joint coordinate formulation because the joint DOFs are used to determine the position and forces of each body. This formulation allows the use of a recursive solution procedure in which Cartesian joint coordinates are calculated by starting from the base body to the terminal bodies (forward path) and the joint reaction forces are eliminated from one body to the next until the base body is reached (backward path). Since constraints are automatically incorporated in the equations of motion from leaf-bodies to the base body, for open-loop systems, only the dynamic equilibrium equations (Eq. (3)) are needed to model the system. For closed-loop systems, however, loop-closure constraint equations (Eq. (2)) must be added. The dynamic equilibrium equations have the same form as Eq. (3), except that now the system matrices are dense because the set of generalized coordinates is minimal. The relative coordinate formulation algorithm was first applied to open-loop rigid multibody systems in Chace [130] and to open-loop FMS in Hughes [133], Book [135], Changizi and Shabana [110], and Kim and Haug [138]. Then, it was extended to closed-loop FMS by adding cut-joint constraints to the equations of motion [111,112,147,148,150]. The closed-loop constraints, as well as prescribed motion constraints, are usually included using Lagrange multipliers. The relative coordinates formulation in conjunction with a recursive solution procedure has been demonstrated to yield near real-time solution for some practical problems (eg, [154,599,600]).

Relative nodal coordinates, along with a recursive solution procedure, have recently been used in conjunction with a corotational-type formulation for FMS which includes beams and rigid bodies in Park *et al* [479] and Cho *et al* [480]. The corotational frame in this case is the frame of the adjacent node to the element. Similar to the floating frame, a recursive algorithm including forward and backward paths is used. A loop-closure constraint equation was added for modeling closed-loop FMS.

Relative coordinates techniques involve the additional step of computing the tree. This can be inconvenient for variable structure FMS and FMS involving contact/impact. In addition, for FMS involving closed loops, the solution depends on the choice of the location of the cut-joint constraint.

2.8.2 Nonlinear inertia effects

As mentioned previously, in the floating frame approach, usually both inertia and internal forces are evaluated with respect to the floating frame. Since the inertia forces are expressed relative to the floating frame, which is a moving frame, they include, in addition to the linear mass times flexible accelerations relative to the floating frame term, three types of terms: nonlinear tangential, centrifugal, and Coriolis inertia forces. These terms couple rigid body acceleration of the floating frame and the flexible body accelerations relative to the floating frame such that a vibration of the body produces a rigid body motion and vice versa.

In the early research on the floating frame approach, the coupling terms were neglected. A rigid body dynamic analysis was first conducted to find the rigid body motion and inter-body reaction forces of the flexible multibody system. Then, for each discrete configuration of the system, the reaction forces are applied to each flexible body to find its flexible deformations. Thus, at each discrete position, the multibody system is assumed to be an instantaneous structure. This approach was adopted in the kineto-elastodynamics of mechanisms (eg, [9,57,58,63]). The effect of the coupling between flexible and rigid body motion becomes more important as the ratio between the rigid body inertia forces and the flexible body inertia forces decrease. This ratio increases by mounting flywheels with high moments of inertia to the axis of the rotating flexible body. Researchers working on kineto-elastodynamics of mechanisms found that adding the coupling terms has very little effect on the response [181,310]. This is because the mechanisms have large flywheels and are stiff closed-loop FMS. For FMS that do not have large flywheels, such as robotic manipulators and space structures, the coupling terms are essential for accurate response prediction.

The importance and need for the rigid-flexible motion coupling were recognized very early in the development of the floating frame approach. Viscomi and Ayre [67] and Chu and Pan [179] derived the partial differential equation governing the motion of the flexible connecting rod of a slider-crank mechanism which includes the inertial coupling terms. Sadler and Sandor [102] and Sadler [178] developed a lumped mass finite difference type nonlinear model for flexible four-bar linkages. Thompson and Barr [316] presented a variational formulation for the dynamic modeling of linkages where Lagrange multipliers are used to impose displacement compatibility at the joints, and some coupling terms are included. Cavin and Dusto [123] derived the governing semi-discrete finite element equations of a single flexible body including the coupling terms using a body mean-axis formulation. The axial deformation was neglected in Viscomi and Ayre [67] and Sadler and Sandor [102], and was included in Chu and Pan [179]. Neglecting the axial deformation means that the centrifugal stiffening effect and the nonlinear inertial coupling terms which involve the axial deformation, are neglected. The effect of these additional terms is negligible for mechanisms with high axial stiffness undergoing relatively slow rotation and small deformations.

The limitation of computational speed and the lack of a standard formulation of the coupling terms between rigid body and flexible body motion made the inclusion of these terms difficult until the late 1970s. Then a series of papers presented floating frame absolute coordinates finite element formulations which include the coupling terms [72,103,106,108,180,181,270]. Floating frame formulations based on relative coordinates which include the coupling terms were presented by Kim and Haug [138] and Ider and Amirouche [111]. Shabana and Wehage [106,180] suggested the current widely used form of the inertia coupling terms. This form can be easily used in conjunction with modal reduction techniques and it clearly identifies the various coupling terms. In this form, the generalized coordinates are partitioned in the following way:

$$q=[q_T \quad q_\theta \quad q_f]^T \tag{7}$$

where subscripts T, θ, and f denote rigid body translation, rigid body rotation, and flexible coordinates, respectively. The corresponding system's mass matrix in Eq. (4) can be written as:

$$M=\begin{bmatrix} M_{TT} & M_{T\theta} & M_{Tf} \\ & M_{\theta\theta} & M_{\theta f} \\ sym. & & M_{ff} \end{bmatrix} \tag{8}$$

The matrix M_{TT} is a constant translational mass matrix which represents the mass of the entire body, M_{ff} is the constant finite element mass matrix, $M_{\theta\theta}$ is the rotary inertia matrix which represents the inertia tensor of the flexible body ($M_{\theta\theta}$ is approximately constant if the body deformations are small, otherwise it is time varying), $M_{\theta f}$ and M_{Tf} are time-varying matrices (which are a function of the generalized coordinates) which represent the inertial coupling between the gross rigid body motion and the flexible deformations, and $M_{\theta T}$ is a time-varying matrix representing the inertial coupling between the rigid body translation and rigid body rotation. The Coriolis and centrifugal forces are quadratic in velocities and are also nonlinear in the generalized coordinates. They are added to Eq. (4):

$$F_c=\dot{M}\dot{q}+\frac{1}{2}\frac{\partial}{\partial q}(\dot{q}^T M\dot{q}) \tag{9}$$

where $\dot{M}\dot{q}$ is the Coriolis force vector and $\frac{1}{2}\partial/\partial q\,(\dot{q}^T M\dot{q})$ is the centrifugal force vector.

Another important nonlinear inertial effect is dynamic or centrifugal stiffening. The centrifugal component of the inertia force acts along the axis of the rotating body causing an axial stress that increases the bending stiffness of the body [55,204,206]. In addition, if this body is connected to other bodies, then the rotation of the other bodies will cause a stiffening effect on the root body because of the transfer of inter-body forces through the joints [111,203,214,221,222]. If a classical beam element is used for the flexible component, the bending deformation is not coupled with the axial deformation, which means that dynamic stiffening is neglected. Many flexible multibody analysis codes developed in the early 1980s had this flaw. Kane *et al* [205] showed that, for a rotating flexible beam undergoing a spin-up maneuver, neglecting the centrifugal stiffening term results in the wrong prediction that the beam diverges during the maneuver. They demonstrated that by using a nonlinear strain-displacement relation, which couples the axial and bending strains, proper stiffening effects are included. This was followed by numerous other studies investigating the dynamic stiffening effect and developing new modeling techniques to accurately incorporate the effect in general FMS (see Table 3). In a finite element formulation, the centrifugal stiffening term is usually included in a nonlinear stiffness matrix K_{NL} that is added to the partitioned equation of motion (see Eq. (2)) yielding the following form for the system stiffness matrix:

$$K=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_L+K_{NL} \end{bmatrix} \tag{10}$$

where K_L is the linear constant stiffness matrix (if a linear constitutive material law is used). K_{NL} is nonlinear and time-varying which may include, in addition to the coupling between axial deformation and transverse bending deformation which gives rise to the centrifugal stiffening effect, quadratic strain-displacement terms which account for moderate flexible deflections (see the succeeding subsection). The use of the nonlinear stiffness matrix K_{NL} makes it difficult to use modal reduction techniques. This is further discussed in the Subsection 2.8.4.

2.8.3 Treatment of geometric nonlinearities

In order to extend the deflection range of a body when the floating frame approach is used, quadratic terms in the strain-displacement relation can be included. In Table 3 publications in which these terms are included are listed. The nonlinear quadratic strain terms are added to the nonlinear stiffness matrix K_{NL} (Eq. (10)). An important effect, which is included by incorporating the axial-bending quadratic strain terms, is the foreshortening effect, which is the shortening of the projected length of a beam relative to its reference straight configuration when it bends. This means that a transverse displacement of a point on the beam gives rise to an axial displacement. In the floating frame approach, because the deformations are superimposed on the rigid body reference configuration, the rigid body length is usually kept constant, which means that foreshortening is neglected. Accounting for foreshortening requires updating the body inertia tensors. Foreshortening becomes more important as the deflection increases.

2.8.4 Modal reduction

A major advantage of using the floating reference frame is that the physical finite element nodal coordinates can be easily reduced using modal analysis techniques based on using a reduced set of eigen-vectors of the free vibration discrete equations of motion as flexible modal coordinates. The reduction is achieved by eliminating the high frequency modes, which carry little energy. Modal reduction offers an efficient way to reduce the number of DOFs with the minimum deterioration in accuracy. Based on the coordinate partitioning strategy suggested in Shabana and Wehage [106,180], modal reduction can be done by using the following transformation for the generalized coordinates:

$$\begin{Bmatrix} q_T \\ q_\theta \\ q_f \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & W \end{bmatrix} \begin{Bmatrix} q_T \\ q_\theta \\ P_f \end{Bmatrix} \tag{11}$$

where I is the identity matrix, W is the modal matrix that consists of a finite set of eigenvectors (up to the eigenvector corresponding to the desired maximum natural frequency) and P_f is the vector of generalized modal coordinates. In many FMS applications, the high frequency modes carry little energy and thus have a negligible effect on the overall dynamic motion of the multibody system. Also, the presence of the high frequency modes increases the stiffness of the equations of motion and requires the use of a small integration time step. So if these modes are eliminated, the gain in computational speed is twofold. First, a larger integration time step can be used. Second, the reduction in the number of flexible DOFs reduces the number of equations of motion that need to be solved. Detailed deformation and stress fields in a flexible body can be calculated using an FEM program in a post-processing stage. This can be done by applying the computed inertia forces in addition to the applied loads and constraints to a detailed FE model of the flexible body [312,601] or by applying the deformations following from the modal coordinates to the FE model [602].

The mode shapes and natural frequencies that are used in modal reduction can be obtained either by modal reduction of a finite element model or by using experimentally identified modes [269]. The ability to use modal reduction (especially experimentally identified modes) is the main factor for the widespread use of the floating frame approach in modeling FMS. Very early in the development of the floating frame approach, modal reduction and normal mode techniques were used in modeling space structures with flexible appendages [52–54,59,233] and in the kineto-elastodynamics of mechanisms [104,232]. Then, later modal reduction was applied to finite element models of general FMS (see Table 3).

Modal reduction can achieve large reductions in computation time only if the body mass and stiffness matrices are constant (ie, are not a function of time or generalized coordinates). The modal reduction, in this case, is performed once at the beginning of the simulation. If the mass or stiffness matrices are not constant, then modal reduction must be performed at each time step, which defeats the purpose of reducing the computation time. If the deflection of the body is small and its angular velocity is low or constant, then the body mass and stiffness matrices are approximately constant with respect to the floating frame. Large deflections introduce quadratic terms in the strain-displacement relations. Large variable angular velocities make the centrifugal stiffening term time varying. Thus large deflections and large variable angular velocities make the stiffness matrix, and hence the natural frequencies and mode shapes of the flexible bodies, nonlinear and time-varying (a function of the flexible body coordinates and angular velocities) [32,204, 257,258]. For example, Khulief [32] showed that the response of the coupler and follower of a four-bar linkage calculated using modal coordinates deviated significantly from that using physical coordinates. Ryu *et al* [43,44] developed a time varying stiffness matrix that can be used to extract time-varying Eigen modes of centrifugally stiffened beams, which can be superposed on the linear Eigen modes. The method, however, requires a modal reduction at each time step.

The nonlinear inertial coupling terms make the inertia tensor of a body nonlinear. However, using the coordinate partitioning technique developed in Shabana and Wehage [106], linear modal reduction techniques can be applied only to the flexible coordinates mass matrix [184,191,236,253,262,271]. In order to allow the floating frame and modal coordinates to be used in problems involving large deflections, several researchers developed a sub-structuring procedure in which each body is divided into a number of sub-structures

[21,167,278,281–283,285,287]. Modal reduction is performed for each sub-structure relative to a frame fixed to it. Thus, in large deflection problems the deflections inside a sub-structure are still small and modal reduction is still valid.

The flexible behavior of a body is dependent on the choice of the component modes since a flexible body can only deform in the space spanned by the selected modes. The calculation and selection of these modes requires experience and judgment on the part of the analyst. This is because the boundary conditions, which are used to calculate the deformation mode shapes, do not usually fit a standard description (such as simply supported, fixed-fixed, or cantilevered) and sometimes the description may be configuration dependent [246,264]. In addition, the choice of the deformation modes depends on the choice of the definition of the floating frame—fixed [282] or moving body axes [275]. Thus, in practical application of modal reduction, the analyst must insure that the experimental or numerical modes used match the boundary conditions of the actual system where the component will be placed [87]. Thus, modal reduction requires experience on the part of the analyst. Several researchers have addressed the issue of the selection of the deformation modes and their relation to the boundary conditions and floating frame definition [109,256,261–266,275,603]. For large FMS, which can involve thousands of components, the modal reduction step may require a very long time from an experienced analyst. Thus, the increase in model preparation time can far outweigh the reduction in computer time.

2.8.5 Governing equations of motion

There are many choices for writing the governing equation of motion of a multibody system. These include: Lagrange's equations, the Hamilton principle, Kane's equations, and Newton-Euler equations. In the first three choices, scalar quantities such as kinetic energy, potential energy, and virtual work are used. In these formulations the nonworking constraint forces are automatically eliminated from the derivation of the equations of motion. This is useful for rigid body dynamic type analyses because it means reducing the number of unknown forces by the number of nonworking constraint forces. However, in FMD the constraint forces are working forces because they cause deformations; therefore all the forms of the governing equations lead to similar semi-discrete equations of motion. In Table 3, papers are classified according the type of governing equations of motion used during the derivation of the semi-discrete equations of motion.

2.9 Summary of the key advantages and limitations of the three frame formulations

The floating frame approach, in conjunction with modal coordinates, is currently the most widely used method for modeling FMS. This is because:

- The floating frame approach provided a direct way to extend rigid multibody dynamics codes for modeling FMS.
- Reduced modal coordinates can be used in conjunction with the floating frame formulation. Mode shapes and frequencies can be either obtained from a finite element model or from experiments. Experimental modal identification is extensively used for transportation vehicles and space structures (eg, [604,605]).
- For small deflections and low angular velocity applications (such as space structures applications), the floating frame formulation, in conjunction with modal coordinates, offers the best mix of speed and accuracy. In the 1970s and 1980s the reduction in computational effort offered by modal coordinates was essential to be able to solve practical problems in a reasonable time.

The corotational and inertial frame approaches share the following advantages over the floating frame approach:

- The translational part of the inertia tensor is linear and constant.
- Kinematic nonlinear effects such as large deflections, centrifugal stiffening, and foreshortening are automatically accounted for. The accuracy of accounting for these effects increases with mesh refinement.

Despite the aforementioned advantages, the corotational and inertial frame approaches have not been widely used for modeling FMS until the early 1990s due to the following:

- The corotational frame approach arose out of research in computational structural dynamics, while the inertial frame approach arose out of research on the large deformation nonlinear finite element methods. The floating frame approach, on the other hand, arose out of research on rigid multibody dynamics, which is conceptually closer to FMD.
- Modal reduction techniques cannot be easily applied with current corotational and inertial frame formulations. Therefore, for small deflection FMS problems, the computation time is generally considerably larger than that of techniques relying on the floating frame and modal reduction. The limited computational speed up to the late 1980s made the corotational and inertial frame approaches unattractive for solving practical FMS problems.
- Rigid body closed loops are difficult to include in a corotational and inertial frame formulation because the optimum solution procedure for rigid body closed loops is fundamentally different from the optimum flexible body corotational or inertial frame solution procedures.
- In practical multibody applications, some components may be very stiff. Those components require very small integration time steps, which make the solution very slow. In a floating frame approach, on the other hand, when modal reduction is used, the stiff modes can be discarded.
- For the inertial and corotational frames, the computation time is the same for small deflection and large deflection problems. This is because the formulation used in modeling large deflections is the same formulation required to account for the large rigid body rotation. Therefore, the small deflection assumption, which is valid in a large number of practical FMS, does not reduce the computation time. In addition, in the inertial frame approach, the computation time is also the same for small strain and large strain problems.

Recent advances have relaxed some of the above difficulties. Some of these advances are:

- Computer speeds have increased by nearly three orders of magnitude since the mid-1980s. At the same time, computer prices have dropped. Thus, the computational cost has considerably decreased, making the corotational and inertial frame formulations economical for more practical FMS applications. In addition, new clusters of massively parallel processors allow fast solution of many practical large FMS.
- There are many commercial codes (eg, DYNA, MSC/DYTRAN, and ABAQUS/Explict) based on the corotational and inertial frame approaches that incorporate rigid components, with the restriction that at least one flexible component must be present in a closed loop. These codes also have a large library of joints such as revolute, prismatic, cylindrical, spherical, planar, and universal joints.
- Multi-time step explicit and hybrid explicit-implicit procedures [489] have been developed to solve stiff problems with disperate time scales at a considerable saving in computer time.

These recent advances, coupled with the advantages of the corotational and inertial frame formulations, have made these formulations very attractive for practical FMS applications. Many researchers recently applied the corotational frame approach to beam-type FMS [31,38,46,47,84–88, 91,453,460] and to shell-type FMS [91,466]. Also, many researchers recently applied the inertial frame approach to beam-type FMS [34–37,39,48–50,96,97,501–503] and to shell-type FMS [514,518,524,528].

3 CONSTRAINT MODELING IN FLEXIBLE MULTIBODY DYNAMICS

Constraints can be divided into three main types: prescribed motion, joints, and contact/impact. The three types can be written in the following compact form:

$$f(q,t)=0 \quad \text{(Prescribed motion)} \tag{12}$$

$$f(q)=0 \quad \text{(Joints)} \tag{13}$$

$$f(q)\geqslant 0 \quad \text{(Contact/impact)} \tag{14}$$

where q is the vector of generalized system coordinates, t is the running time, and f is the generalized constraint function. These constraints give rise to constraint reaction forces that are normal to the direction of motion. In addition, they can produce friction, damping, and elastic forces in the direction of motion. In the following subsections, the various FMD techniques for modeling joints, prescribed motion constraints, and contact/impact are reviewed.

3.1 Joint and prescribed motion constraints

Prescribed motion constraints and joints are modeled by using constraint equations which relate some of the generalized coordinates in such a way as to allow only the kinematic motion allowed by the constraint or joint. The methods for incorporating general constraints into the differential equations of motion of FMS, include:

- Lagrange multiplier method
- Penalty method
- Augmented Lagrangian method
- Relative coordinates method
- Special methods for hinge Joints
- Internal element constraints

Table 6 shows a partial list of papers where the various methods for constraint enforcement are used.

3.1.1 Lagrange multipliers

In the Lagrange multiplier technique, constraint reaction forces F_R (see Eqs. (2,3)) of the form:

$$F_R=-\frac{\partial\Phi^T}{\partial q}\lambda \tag{15}$$

are added to the global equations of motion. In Eq. (15), $\partial\Phi/\partial q$ is the Jacobian of constraint equations and λ is the vector of Lagrange multipliers. Lagrange multiplier method is used to incorporate holonomic and non-holonomic constraints in rigid multibody systems.

The method was applied to FMS using the floating frame approach in Thompson and Barr [316], Song and Haug [103], and Blejwas [368] and is currently the most widely used method for incorporating constraints in the floating frame formulation. It is also used in the relative joint coordinates formulation to enforce loop-closure constraints. Equations (2) and (3), which are the governing semi-discrete equations of motion of the FMS, form a system of DAEs of size $6N+m+c$, where N is the total number of bodies, m is the total number of elastic DOFs, and c is the total number of Lagrange multipliers [103]. For the absolute coordinate formulation, the number of Lagrange multipliers is equal to the total number of constraints. In this case, the equations of motion have the maximum number of coordinates and thus the formulation is called the augmented formulation. The number of DOFs can be reduced to $6N+m-c$ independent coordinates prior to the solution procedure by eliminating the dependent coordinates and associated Lagrange multipliers. A variety of methods have been developed to perform this reduction and obtain an expression of the dependent DOFs in terms of the independent DOFs. These include: the orthogonal complement to the constraint matrix (zero eigenvalue theorem) [5,371,606–610], the singular value decomposition method [72,611,612], coordinate partitioning methods using LU factorization [147,613–618], and up-triangular decomposition of the constraints Jacobian matrix using Householder iterations [113,619–622]. Using the relative coordinate formulation, this reduction is automatically obtained for tree type FMS [111,112,157]. For closed-loop FMS, a Lagrange multiplier is needed for each loop-closure constraint. The Lagrange multiplier method has also been used with the inertial frame approach for modeling revolute joints [505,560], universal joints [623], and prismatic joints [624].

The Lagrange multiplier method has the advantage that the constraints are satisfied exactly (within the accuracy of the numerical iterations) and that the equations of motion for arbitrary configuration FMS including holonomic and non-

Table 6. Classification of a partial list of references on constraint enforcement methods

Method	Floating frame	Corotational frame	Inertial frame
Lagrange multiplier	Thompson and Barr [316], Song and Haug [103], Blejwas [368], Shabana and Wehage [106,180], Samanta [628], most references after 1980.	Wu *et al* [87,457], Housner [46], Housner *et al* [47], Devloo *et al* [463].	Bauchau *et al* [505,560], Bauchau [623,624], Ibrahimbegovic *et al* [512].
Penalty	Serna [373], Bayo *et al* [177].	Devloo *et al* [463].	Avello *et al* [39], Orden and Goicolea [533], Orden and Goicolea [533], Wasfy and Noor [528].
Augmented Lagrange			Park *et al* [502,546], Cardona *et al* [629], Cardona [347], Downer *et al* [36].
Relative coordinates	**Open-loop multibody systems (tree configuration).** Hughes [133], Book [135], Singh *et al* [107], Usoro *et al* [136], Benati and Morro [137], Changizi and Shabana [110], Kim and Haug [138], Han and Zhao [139], Shabana [140,142]. **Closed and Open-Loop multibody systems.** Kim and Haug [147], Ider and Amirouche [111,112], Keat [148], Nagarajan and Turcic [149], Lai *et al* [150], Ider [151], Pereira and Proenca [152], Nikravesh and Ambrosio [153], Hwang [155], Hwang and Shabana [117,156], Shabana and Hwang [116], Jain and Rodriguez [154], Amirouche and Xie [144], Verlinden *et al* [157], Tsuchia and Takeya [158], Pereira and Nikravesh [118]. Pradhan *et al* [160].	**Closed and Open-Loop multibody systems** Park *et al* [479], Cho *et al* [480].	
Modeling hinge joints by sharing a node	Pan and Haug [255].	Yang and Sadler [84], Hsiao and Jang [29], Wasfy [85,86,460,630], Wasfy and Noor [91], Elkaranshawy and Dokainish [31], Iura and Atluri [453].	Simo and Vu-Quoc [34,50].
Internal element constraints			Ibrahimbegovic and Mamouri [511], Ibrahimbegovic *et al* [512], Jelenic and Crisfield [627], Iura and Kanaizuka [598].

holonomic constraints can be constructed systematically. A disadvantage of the method is that it leads to a system of DAEs with a non-minimal set of coordinates $6N+m+c$. Also, zero terms are introduced on the diagonal of the equivalent nonlinear stiffness matrix (see Subsection 4.1.1), which considerably increase its stiffness and required solution effort. Coordinate reduction methods for obtaining the $6N+m-c$ set of coordinates require additional computational effort and often produce a stiffer system of DAEs that is harder to solve.

3.1.2 Penalty method

In the penalty method, the reaction forces associated with the constraints can be written as (see Eq. (2)):

$$F_R=\frac{\partial\Phi^T}{\partial q}\,\alpha\,\frac{\partial\Phi}{\partial q} \tag{16}$$

where α is a diagonal matrix that contains the penalty factors for each constraint equation. The method has the disadvantage that the constraint equations are not satisfied exactly and that large αlead to stiff equations; however, it avoids the difficulties of the Lagrange multiplier approach of solving a system of DAEs. The penalty method was used in Bayo *et al* [177] and Avello *et al* [625] for modeling joints in rigid multibody systems. It was used in conjunction with the inertial frame approach for flexible and rigid multibody systems in Avello *et al* [39], Goicolea and Orden [532], and Wasfy and Noor [528]. Penalty springs can be used to connect components with incompatible nodal interfaces and to represent the shape and stiffness of joints [626].

Following is a systematic way for choosing the stiffness of the penalty spring. If the joint stiffness is on the order of the stiffness of the other components of the FMS, then the penalty spring stiffness can be set equal to the joint stiffness. In this case, the method is physically appropriate. Often, however, the joint stiffness is several orders of magnitude higher than the stiffness of other components/elements. In this case, the stiffness of the penalty spring can be chosen to be equal to the stiffness of the stiffest element in the system. The constraint will not be satisfied exactly, however, this choice will insure that the error introduced due to the penalty spring will be of similar magnitude to the discretization error. Also, this choice insures that the penalty spring does not make the system stiffer (thus harder to solve) than it already is. Thus, in summary, the stiffness of the penalty spring should be equal or less than the physical joint stiffness.

The penalty method can also be used to impose the rigidity constraint of a rigid body [532,539–541]. Goicolea and

Table 7. Classification of a partial list of references on the various types of joints

Joint Type	Floating frame	Corotational frame	Inertial frame
2D revolute	All references on planar FMS.	Most references on planar FMS.	Most references on planar FMS.
3D revolute	Shabana [140], Cardona *et al* [629], Huang and Wang [190].	Most references on spatial FMS.	Most references on spatial FMS.
Spherical	Most references on spatial FMS.	Most references on spatial FMS.	Most references on spatial FMS.
Universal			Bauchau [623], Jelenic and Crisfield [627].
Cylindrical	Shabana [21,140].		Orden and Goicolea [533], Bauchau [624].
Prismatic	Chu and Pan [179], Buffinton and Kane [338], Pan [339], Pan *et al* [340,341], Hwang and Haug [342], Shabana [21,140], Azhdari *et al* [354], Gordaninejad *et al* [343], Buffinton [344], Al-Bedoor and Khulief [345], Verlinden *et al* [157], Fang and Liou [194], Theodore and Ghosal [346].		Bauchau [624], Orden and Goicolea [533], Wasfy and Noor [528] **Axially moving beam:** Downer and Park [503], Vu-Quoc and Li [561].
Planar			Orden and Goicolea [533].
Lead screws	Chalhoub and Ulsoy [639].		
Gears	Amirouche *et al* [640].		Cardona [347].
Cams	Bagci and Kurnool [348].		Cardona and Geradin [638].

Orden [532] modeled rigid bodies by using multiple points on the body connected using stiff penalty springs.

3.1.3 Augmented Lagrangian method

The augmented Lagrange method combines both the Lagrange multiplier and the penalty methods in order to reduce the disadvantages of both methods. By introducing a penalty spring whose stiffness is comparable to the stiffness of other components of the FMS, the number of iterations and effort required to solve the system of DAEs can be reduced. The constraint is satisfied exactly at the end of each solution time step. Downer *et al* [36] and Park *et al* [502,546], used the augmented Lagrange method with the inertial frame approach to model general holonomic and non-holonomic constraints. A coordinate partitioning scheme was used in Park *et al* [502] to eliminate the Lagrange multipliers.

3.1.4 Relative coordinates

For open-loop FMS (tree configuration), joint constraints can be automatically satisfied using the floating frame and the relative coordinate formulation (see Table 2). As mentioned in Subsection 2.8.1, the coordinates of a body (child body) in a chain of bodies are expressed in terms of the coordinates of the previous body (or parent body) in the chain and the DOFs of the joint connecting the two bodies. Thus, the joint constraints are automatically incorporated from the root body to the tip body. However, closed loops and prescribed motion constraints still need the addition of constraint equations. These types of constraints are usually enforced using the Lagrange multiplier technique [111,112,153,157]. The Lagrange multipliers can then be eliminated in order to obtain a minimal set of coordinates [153,240].

3.1.5 Special method for rotational hinge joints

Rotational hinge joints constrain the translational DOFs between two bodies and allow some rotational motion. They include: spherical, universal, and revolute joints. For the inertial and corotational frames, hinge joints can be modeled by letting two bodies share a node and then constrain the relative rotation at that node as required by the joint [31,50,86,453,460]. The Lagrange multipliers or penalty methods can be used to impose the rotation constraints, but are not required for imposing the translation constraints.

3.1.6 Internal element constraints

Recently, a type of methods for enforcing constraints that do not require penalty parameters or Lagrange multipliers have been developed. The methods are based on explicitly imposing the constraints into the element arrays and the time-integration solution procedure. Ibrahimbegovic and Mamouri [511] incorporated revolute, prismatic, universal, and rigid joints into a spatial geometrically exact beam element. Also, in Jelenic and Crisfield [627], a spatial geometrically beam element with an *end release* which introduces the joint kinematics in the element formulation was used to model revolute, prismatic, and universal joints. Iura and Kanaizuka [598] developed a similar approach for translational joints by using a modified shape function in an element-free Galerkin formulation. The method has the advantage of not requiring additional variables or additional algebraic equations. However, it requires reformulating the existing elements.

3.2 Joint types

Table 7 shows a classification for the various joint models used and developed in the literature. These are:

Revolute, Spherical, and Universal Joints. These joints connect two bodies at a point. All the translational displacement components at the joint are equal for the two bodies while some rotational freedom is allowed, thus these joints are also called hinge joints. The revolute joint leaves only one rotational DOF free and constrains the remaining two, the universal joint leaves two rotational DOFs free and constrains one, and the spherical joint leaves all three rotational DOFs free. The revolute joint is the most common type of joint and thus it has been used in most multibody dynamics studies. For a revolute joint in 3D, two constraints are added in order to constrain the relative rotation between the two bodies to the plane of the revolute joint. Clearances in 2D revolute joints were addressed by Dubowsky and Freudenstein [631], Winfrey *et al* [632], Dubowsky and Gardner [69,70], Soong and Thompson [633], and Amirouche and Jia [634]. Lubrication effects were modeled in Liu and Lin [635] and Bauchau and Rodriguez [636] by solving the Reynolds lubrication equation.

Prismatic, Planar, and Cylindrical Joints. These joints connect a point on a body to a line or surface on another body. Prismatic joints allow only one translational DOF and constrain the two remaining translation DOFs as well as the three rotation DOFs. Planar joints allow two translational DOFs and constrain the remaining translation DOF as well as the three rotation DOFs. Cylindrical joints allow only one translational DOF along an axis and one rotational DOF around that axis and constrain the remaining DOFs. Prismatic joints are used in slider-crank mechanisms which are present in many machines, most notably internal combustion engines.

Gears. Gears are devices for the transmission of rotary motion from one shaft to another. The general type of gears is 3D gearing where the two shafts are not necessarily parallel. All kinds of gears are a particular case of 3D gearing: eg, spur gears, bevel gears, hypoid gears, worm gears, etc, Cardona [347] developed a methodology for modeling general gears within an inertial frame formulation using a set of holonomic and non-holonomic constraints. Two nodes, one at the center of each gear, are used to model the gear joint.

Cams. Cams are devices for the transformation of rotary motion to a desired linear motion. Cams are most notably used in internal combustion engines to control the air intake and exhaust from the cylinders. They are also widely used in industrial machines. Bagci and Kurnool [348] modeled cam driven linkages using the theory of elasto-dynamics in which the linkage is considered as an instantaneous structure at each snapshot of motion. The periodic response of a cam-driven valve train with clearances was studied in Wang and Wang [637]. The dynamic response of cams, including intermittent motion and Coulomb friction, was studied by Cardona and Geradin [638].

Lead Screws. Lead screws are devices for the transformation of a large rotary motion to a much smaller linear motion, thus gaining a large mechanical advantage. Chalhoub and Ulsoy [639] used the floating frame approach to model a flexible robot driven by a lead screw.

3.3 Treatment of contact/impact

Contact/impact modeling is used in a number of application areas including: crash-worthiness analysis, metal forming, and multibody dynamics. A review article on contact/impact by Zhong and Mackerle [641] includes about 500 references. While some publications deal exclusively with one application area, other publications develop general contact/impact methods. Some FMD applications which involve contact/impact are: joint clearances [636], intermittent motion mechanisms [333,334], clutches [552], belt drives [551,553], variable kinematic structure mechanisms (involving addition or deletion of joints), robot grasping, and docking and assembly of space structures (variable mass FMS involving mass capture/release) [335,336,642]). There are four physical conditions present in a contact/impact problem:

1) The displacements of the contact point on the first body and the corresponding contact point on the second body must be such that the two bodies do not overlap.
2) The reaction forces at a contacting point on the first body and the corresponding point on the second body must be equal in the static contact limit.
3) The total momentum and energy of the two impacting bodies must be conserved in case there is no other source of energy or momentum gain or dissipation.
4) In case there is a relative motion between the two contacting bodies, a friction force in a direction tangential to both contacting surfaces must be added. The magnitude of this force is a function of the normal reaction force between the two bodies. The most widely used friction model is the Coulomb friction model in which the friction force is proportional to the normal reaction force.

Contact/impact modeling methods attempt to model the contact/impact phenomena while satisfying the above conditions. In order to satisfy condition 1, a method for detection when contact occurs—contact searching—is needed. Zhong and Mackerle [641] classify contact searching algorithms according to: master-slave algorithms [486] and hierarchical-territory algorithms (HITA) [641,643–645]. In the HITA, four types of hierarchies can be used: the contact bodies, the contact surfaces, the contact segments, and the contact nodes. The territory of each hierarchical branch is used to detect contact, thus speeding up contact searching by eliminating higher level branches without having to search through the lower level branches.

Once contact is detected, two main types of methods have been used to satisfy conditions 1 and 2. These are: contact force based methods and momentum-impulse methods. Contact force based methods can be further divided into: the penalty method, the Lagrange multipliers method, and the augmented Lagrange method [641]. Momentum-impulse methods can be divided into: global and local methods. In this section, the contact/impact modeling methods that are used in conjunction with FMD applications are reviewed. Literature classification for the various FMS Contact/Impact modeling methods are shown in Table 8 and a brief explanation of each method will be given in the subsequent subsections.

3.3.1 Penalty method

In the penalty method, the contact pressure is assumed to be equal to the amount of penetration times a penalty parameter. This is equivalent to introducing a *penalty spring* between the contacting points. A penalty damper can also be used. The same procedure described in Subsection 3.1.2 for selecting the penalty stiffness and damping for joints can be used in contact/impact modeling (eg, [641,646]). A physical contact force model such as Hertzian contact force can also be used [647–649]. In Khulief and Shabana [650] the stiffness

Table 8. Classification of a partial list of references on contact/impact modeling methods

Contact/Impact method	Floating frame	Corotational frame	Inertial frame
Penalty/physical contact force	Khulief and Shabana [650], Wu and Haug [281], Huh and Kwak [658], Ko and Kwak [659,660], Amirouche *et al* [661], Dias and Pereira [662]. **Effect of Modal Reduction** Escalona *et al* [649]. **Friction Model** Haug *et al* [663], Pereira and Nikravesh [118], Lankarani and Nikravesh [664].		Lee *et al* [665], Lee [666,667], Osmont [668], Sheth *et al* [669], De la Fuente and Felipa [670], Ibrahimbegovic and Wilson [671], Hunek [672], Shao *et al* [673], Huang and Zou [674], Laursen and Simo [529], Qin and He [675], Laursen and Chawla [676], Bauchau [648], Leamy and Wasfy [551,552], Bottasso and Trainelli [677].
Lagrange multiplier	Haug *et al* [663], Wu and Haug [281], Jia and Amirouche [678].	Belytschko and Neal [679], Belytschko [490].	Taylor and Papadopoulos [680], Sha *et al* [681], Wriggers *et al* [682], Bauchau [651].
Global momentum conservation	Khulief and Shabana [333], Bakr and Shabana [653], Rismantab-Sany and Shabana [654], Hsu and Shabana [683], Gau and Shabana [684,685], Yigit *et al* [655,656], Lankarani and Nikravesh [686], Kovecses *et al* [337], Marghitu *et al* [687]. **Effect of Modal reduction** Palas *et al* [657]. **Coulomb Friction** Zakhariev [688].		
Local momentum conservation		Wasfy [85,630], Wasfy and Noor [642].	

and damping coefficients were determined using a momentum balance approach. In practice, for contact between stiff bodies, a large penalty stiffness is used. The larger the value of the penalty stiffness, the more the non-penetration condition is satisfied, but the smaller the required solution time step.

Coulomb friction can be also modeled using a penalty approach where, for small relative tangential velocities between the two bodies, the friction force is proportional to the tangential velocity, up to the Coulomb friction force [551,651]. The larger the value of the proportionality constant, the closer the friction model is to the Coulomb friction law. The penalty contact method, along with this approximate penalty Coulomb friction law, was used to accurately model the dynamic response of belt drives including accurate prediction of the belt stick and slip arcs over the pulleys [551,553].

The penalty method can be used to model intermittent motion mechanical elements. For example, in Leamy and Wasfy [552] a one-way clutch element between two pulleys was used in which the transmitted torque in the clutch transmission direction is equal to a penalty parameter multiplied by the relative angular velocities between two pulleys and zero in the opposite direction.

3.3.2 *Lagrange multiplier and augmented Lagrange methods*

In the Lagrange multiplier method, Lagrange multipliers are introduced in the variational form of the governing equations. Then, constraints are added between nodes in contact to force them to have the same displacement. Lagrange multipliers associated with a constraint represent the contact force. The Lagrange multiplier method is suitable for contact between very stiff bodies. It eliminates the need for an arbitrary large penalty parameter at the expense of adding an extra solution variable—the Lagrange multiplier.

As in the augmented Lagrangian method for joints, both a penalty parameter and a Lagrange multiplier can be used in the contact constraint equation. The penalty parameter reduces the number of iterations required to solve the system equations.

3.3.3 *Global momentum/impulse methods*

In contact force based approaches, a normal reaction force between the two impacting surfaces can be readily calculated. Momentum/impulse methods, on the other hand, predict the jump discontinuities in the system velocities and internal reaction forces as a result of the impact using momentum and impulse conservation equations. Momentum-impulse based methods are well established for impact of rigid bodies (eg, [652]); however, they have only been recently applied to impact of flexible bodies. In Khulief and Shabana [333,334], Bakr and Shabana [653], and Rismantab-Sany and Shabana [654], the generalized impulse momentum equations were used to predict the jump discontinuities in the velocities and joint reaction forces of intermittent motion FMS. The momentum-impulse method was applied to all the generalized coordinates of the two impacting flexible bodies. In Rismantab-Sany and Shabana [654], the convergence of the series solution obtained by solving the generalized impulse momentum equations was used to prove the validity of the approach. In Yigit *et al* [655,656] the validity of the approach was verified experimentally using a flexible rotating beam impacting on a rigid surface. For methods based on the floating frame approach and modal reduction, contact/impact introduces jump discontinuities in the system natural frequencies and mode shapes [336]. The influence of contact/impact on the choice of the reduced modes was studied in

Table 9. Classification of a partial list of references on explicit and implicit solution procedures

Procedure Type	Floating frame	Corotational frame	Inertial frame
Iterative-implicit	Song and Haug [103], Shabana and Wehage[106], Bakr and Shabana [128], Rismantab-Sany and Shabana [701], Shabana [21],Haug and Yen [617], Fisette and Vaneghem [618], Simeon [689].	**Semi-Implicit with Newton Iterations** Housner [46,47], Hsiao and Jang [29,437], Hsiao *et al* [438], Hsiao and Yang [444], Elkaranshawy and Dokainish [31], Banerjee and Nagarajan [448], Devloo *et al* [463].	Nagarajan and Sharifi [537], Simo and VuQuoc [34,49,50], Cardona and Geradin [35], Geradin [702], Bauchau *et al* [505,560], Bauchau and Theron [703], Ibrahimbegovic and Al Mikdad [37].
		Energy conserving: Crisfield and Shi [89,90], Galvanetto and Crisfield [445].	**Energy Conserving:** Simo and Tarnow [518], Simo *et al* [691], Stander and Stein [497], Ibrahimbegovic and Al Mikdad [704], Orden and Goicolea [533], Ibrahimbegovic *et al* [512], Borri *et al* [513], Bauchau *et al* [524]. **Energy Decaying:** Bauchau [623], Bauchau and Hodges [508], Bauchau *et al* [524], Borri *et al* [513].
Explicit	Metaxas and Koh [173].	Flanagan and Taylor [475], Wasfy [85,86,460], Wasfy and Noor [91], Iura and Atluri [453].	Park *et al* [502], Downer *et al* [36], Wasfy [514], Leamy and Wasfy [551,552].
Implicit-Explicit			Park *et al* [502,546], Lim and Taylor [536].

Palas *et al* [657]. The global momentum method has an inherent assumption that the impact propagates in the flexible body at an infinite speed. This assumption is valid for stiff bodies and is not valid for highly flexible bodies.

3.3.4 Local momentum/impulse conservation methods

This technique is based on the use of the rigid body impact modeling tools, namely, conservation of momentum and the restitution equations as local velocity constraints. This technique was presented in Wasfy [630] and Wasfy and Noor [642]. The restitution and conservation of momentum equations (which are equivalent to the energy and momentum conservation equations in case there is no friction between the contact surfaces) are used as local postimpact velocity constraints on the impacting nodes. So, in this approach, contact is considered to be a local phenomenon in which only the motion of the impacting node is directly altered by the impact. The motion of the rest of the finite element model is indirectly altered due to the transfer of the impact effect through internal (structural) forces. The contact force between the surfaces is modeled by the internal forces in the contact region. Frictional effects can be modeled by introducing two restitution coefficients, one in the normal impact direction and one in the tangential impact direction. Unlike impact modeling of rigid bodies, the restitution coefficients are not used to model the energy loss in the body as a whole (this is left to the internal material damping set off by the large deformation rates caused by the impact) or to model energy dissipation as sound and heat due to impact and friction; they only model the local friction force effect at the contact point.

4 SOLUTION TECHNIQUES

In this section, implicit and explicit solution procedures that are used to solve the semi-discrete equations of motion along with the constraint equations (Eqs. (2 and 3)) are reviewed. Also, some of the methods used to enhance the speed and accuracy of the solution procedure and the numerical model are reviewed. These methods are: recursive solution procedures, multi-time step methods, parallel computational strategies, object-oriented strategies, computerized symbolic manipulation, adaptive approximation strategies, and methods for assessing the effects of uncertainties.

4.1 Solution procedures

4.1.1 Implicit solution procedures

In implicit solution procedures (see Table 9), a solution for the system displacements that simultaneously satisfies the equations of motion and constraints is sought at each time step given the solution at the previous time step. Since the equations are nonlinear, Newton-Raphson equilibrium iterations are performed to guarantee that an equilibrium solution is reached at each time step [40–42,530]. A typical solution algorithm is summarized in the following three equations:

$$\{q^*\}^{(1)}_{t+\Delta t}=\{q^*\}_t \tag{17a}$$

$$[K^*]^{(k)}_{t+\Delta t}\{\Delta q^*\}^{(k+1)}_{t+\Delta t}=\{\Delta f^*\}^{(k)}_{t+\Delta t} \tag{17b}$$

$$\{q^*\}^{(k+1)}_{t+\Delta t}=\{q^*\}_t+\{\Delta q^*\}^{(k+1)}_{t+\Delta t} \tag{17c}$$

where t is the running time, Δt is the time step, (k) is the iteration number, and q^* is the vector of generalized coordinates. $[K^*]$ and Δf^* are the equivalent tangent nonlinear stiffness matrix and the vector of equivalent generalized forces. $[K^*]$ and Δf^* are functions of $\{q^*\}^{(k-1)}_{t+\Delta t}$ and the system stiffness, damping, and inertia forces. Equation (17*b*) also includes algebraic equations for the prescribed motion, joint, and contact constraints. The iterations start by setting the value of the generalized coordinates at the first iteration of the next time step $\{q^*\}^{(1)}_{t+\Delta t}$ to be equal to the value of the generalized coordinates at the previous time step $\{q^*\}_t$ (Eq. (17*a*)). The equations of motion are linearized, by neglecting the quadratic Δ terms, at the configuration at time step $t+\Delta t$ and cast in terms of a linear system of algebraic equations (Eq. (17*b*)). This system of equations is solved for Δq^* using Gauss elimination, LU factorization, or the conjugate gradient method. A new estimate of the generalized coordinates is calculated using Eq. (17*c*) and used to calculate a new equivalent tangent stiffness matrix and the equivalent force vector, which are in turn plugged back into Eq. (17*b*). The iterative procedure is repeated until the maximum error between iterations is less than a certain tolerance. For multibody dynamics problems, the solution time and, thus, the number of time steps is large compared to other fields (such as metal forming and crash-worthiness analysis). Thus, the iterative solution tolerance must be set at a small value, which means that a large number of iterations will be required. This is because any error admitted into the solution at a time step will affect the time evolution of the solution in a path-dependent way [530].

Implicit solution procedures are unconditionally stable. However, the time step should be at least an order of magnitude smaller than the smallest natural period that needs to be resolved. An advantage of implicit solution procedures over explicit procedures is that the time step can be much larger than the smallest natural period of the system, which can be very small for very stiff systems. Modes with a natural period of the same order or smaller than the chosen time step are not accurately modeled. Therefore, some experience is needed, when using an implicit solution procedure, in choosing a time step that provides a response within engineering accuracy.

In the evaluation of $[K^*]$, a time integration formula is needed. The most widely used formulas are: the Newmark method [29,31,37,47,437,438,448,618], Runge-Kutta method [30,623,689], Gear's algorithm [84,103, 690], or more generally, backward differentiation formulas. The Newmark method is simple, fast, and unconditionally stable for linear problems, however it has been shown to be unstable for large rotation nonlinear problems [89,497,518,691]. Gear's algorithm and backward differentiation formulas are particularly suited to DAEs since they can be tuned to be stable for stiff equations [690]. The generalized Alpha-method includes a parameter for filtering frequencies above a certain level [480,692]. Geometric integration relies on differential geometry and Lie group theory to achieve total energy, linear momentum, and angular momentum conservation [512,513,518,691]. Some researchers found that the energy conserving schemes can produce non-physical high frequencies in the internal stresses, especially when material damping is present [524,623]. This is due to the fact that the chosen time step is generally at least two orders of magnitude larger than the smallest characteristic time in the problem. The unmodeled high-frequency modes produce the non-physical response. Geometric integration energy decaying schemes were developed based on various numerical integration techniques such as Runge-Kutta and finite difference (eg, [513,524,560,623,693]), which allow filtering the high frequencies by gradually reducing the total energy in a controlled fashion.

There is a very close relationship between the solution methods and the constraints modeling methods. The floating frame approach is usually used in conjunction with the Lagrange multiplier method for imposing the constraints. Two methods are used to include the constraint equations in Eq. (17*b*), namely: the direct method and methods based on reduction of the dependent coordinates. In the direct method, the constraint equations are directly added to Eq. (17*b*) [103,128,694]. The direct method leads to a maximal number of coordinates. The resulting equivalent stiffness matrix $[K^*]$ is generally a sparse matrix. The sparsity of the system equations is computationally advantageous because it has been shown that it is usually more efficient to solve a large system of sparse equations rather than a smaller system of dense equations [695]. But in order to take advantage of the equations sparsity, sparse matrix storage and decomposition must be used. It is inefficient to store and decompose a sparse matrix using a 2D array. The most commonly used method of storing sparse matrices is to store the row and column indices and the value of each nonzero entry of the matrix. A sparse Gauss elimination or LU decomposition can then be performed [695]. Many commercial packages based on the floating reference frame and absolute coordinates (eg, ADAMS and DADS) take advantage of the sparsity of the equations by using sparse matrix techniques [696]. Pan and Haug [379] developed an inertia lumping technique for reducing off-diagonal coupling (ie, increasing the sparsity) of $[K^*]$.

Alternatively, in methods based on reduction of the dependent coordinates, the number of DOFs is reduced to $6N+m-c$ independent coordinates prior to the solution procedure by identifying the dependent coordinates and expressing them in terms of the independent coordinates using a variety of techniques (see Subsection 3.1.1). This results in a minimal number of coordinates and dense system equations. The computational advantage gained by the reduction in the number of coordinates is generally offset by the following:

- The characteristic matrices are denser.
- The nonlinearity of the equations is increased.
- The reduction routine requires a matrix factorization at each time step [21,140].

The floating reference frame with relative coordinates also leads to a dense, strongly coupled equivalent stiffness matrix. But, recursive solution procedures (see Subsection 4.2.1) can be used.

Similar to the floating frame, a major issue in an implicit solution procedure based on the corotational or inertial frames are incorporating the constraint equations into Eq. (17*b*). The various techniques for incorporating the constraints are discussed in Subsection 3.1.

4.1.2 *Explicit solution procedures*

In explicit solution procedures [697], a solution for the nodal accelerations that satisfies the equations of motion and constraints is sought at each time step. If a lumped mass matrix is used, then the system's equations of motion are uncoupled at each time step and they can be directly solved for the nodal accelerations. A typical explicit algorithm starts by evaluating the vector of internal forces (f_{internal}) from the known nodal positions and velocities at time step t. Then, internal forces are added to the external forces f_{external}. The equations of motion are then directly used to calculate the accelerations at time step $t+\Delta t$:

$$\ddot{x}_{t+\Delta t} = M^{-1}(f_{\text{internal}} + f_{\text{external}})_t \tag{18}$$

A time integration formula such as the trapezoidal rule is used to integrate the acceleration into the velocities and positions at time step $t+\Delta t$. Equilibrium iterations can be performed within a time step to improve the stability and increase the critical time step [85,91]. Two equilibrium iterations correspond to predictor-corrector type algorithms. As the number of equilibrium iterations increase, the algorithm approaches an iterative-implicit conjugate gradient algorithm.

Explicit temporal integration techniques are only conditionally stable because the time step must be smaller than the equation's characteristic time. If the same time step is used for the entire FMS, then that time step must be smaller than the smallest natural period of all finite elements. This imposes a severe time step restriction and generally means that a very large number of time steps is needed to obtain the dynamic response of practical FMS. On the other hand, the advantages of explicit solution procedures are:

- All the system modes are accurately resolved.
- Physical material damping does not produce non-physical high frequency oscillations in the response as in implicit methods, but actually helps damp out the high frequencies.
- The number of arithmetic operations at each time step is only $O(N)$, where N is the number of DOFs. This is in contrast with implicit solution procedures, which require at least $O(N^2)$ number of arithmetic operations per time step due to matrix decompositions. Thus, there exists a critical N above which explicit procedures are computationally more efficient than implicit procedures.
- They are *embarrassingly* parallel because all the equations of motion are decoupled at a time step (see Subsection 4.2.3).

Explicit solution procedures were first used for transient analysis of large structures. They were applied to nonlinear structural dynamics using the corotational formulation in Belytschko and Hsieh [45], Belytschko *et al* [698], Hughes and Winget [481], Flanagan and Taylor [475], and Rice and Ting [439]. They are also used for contact/impact large deformation structural dynamics and crash-worthiness analysis (eg, [681,699,700]). Explicit solution procedures are well suited for problems involving high deformation rates and high-speed wave propagation such as automobile crash-worthiness analysis. Table 9 lists the references where explicit solution procedures are used for FMS.

A variety of time integration formulas are used with explicit solution procedures such as: central difference [439], Newmark method [85,86,91,460]), and fourth order Runge-Kutta method [453].

The incorporation of constraints in explicit solution procedures depends on the type of constraint. Hinge-type joints do not introduce extra constraint equations because they can be modeled by sharing a node between two bodies [34,50,460], thus they do not require any special treatment. For prescribed motion constraints, the constraint equations can be executed within the explicit iterations to enforce their satisfaction [85,86]. For joints and contact/impact the, following constraint enforcement methods can be used:

- The penalty method [528,551,552]
- The augmented Lagrangian method in conjunction with a separate implicit solution for the Lagrange multipliers [36,502,546]
- Lagrange multiplier method in conjunction with a conjugate gradient iterative projection algorithm [681]

4.1.3 *Explicit-implicit solution procedures*

Recognizing the advantages of explicit methods for flexible multibody systems undergoing high speed/acceleration and that of implicit methods in dealing with stiff DAEs, Lim and Taylor [536] suggested using an explicit integrator for flexible bodies and an implicit integrator for rigid bodies along with a node based explicit-implicit partitioning for interface elements.

4.2 Enhancements of the computational process

4.2.1 *Recursive solution procedures*

Recursive formulations are used in conjunction with the floating reference frame and relative coordinates. The relative joint variables describe the large translation and rotation between successive system components. The recursive solution procedure consists of two main steps [135], 1) the recursive evaluation from base to tip of the body position, velocity, and acceleration in terms of all the previous bodies in the chain, and 2) the recursive evaluation from tip to base of the internal forces and moments. Using the relative coordinate formulation, the joint constraints are automatically included for open-loop systems with no prescribed motion constraints. Thus, the resulting equations for open loops do not include Lagrange multipliers and consist of a minimum set of independent coordinates. The gain in computational speed is thus twofold. First, the recursive solution algorithm is $O(N)$ [147,150,154], where N is the number bodies, which means that the computational time grows only linearly with the number of rigid bodies. Second, a minimal set of equations of motion is used. The algorithm was applied to open-loop rigid multibody systems in Chace [130], Wittenburg [131] and Roberson [132], and to open-loop FMS in Book [135], Changizi and Shabana [110], Kim and Haug [138], Shabana [140,141], Shabana *et al* [142], and Amirouche and Xie [144]. Then, it was extended to closed-loop FMS by adding cut-joint constraints to the equations of motion [111,112,116,117,147,148,150,151–156,158]. The cut-joint closed-loop constraints, as well as prescribed motion constraints, are usually included using Lagrange multipliers along with Newton type equilibrium iterations (eg, [111,112,147]). The recursive algorithm is, in most studies, applied to hinge type joints (revolute and spherical joints) (eg, [154]). It was also applied to prismatic and cylindrical joints in Shabana *et al* [142]. In Hwang [155], Shabana *et al* [142], and Hwang and Shabana [117,156], a recursive procedure for decoupling the elastic and rigid body acceleration while maintaining the coupling between rigid body and flexible body motion was developed. The relative coordinates formulation, in conjunction with a recursive solution procedure, has been demonstrated to yield a near real-time solution of the FMS dynamic response in Bae *et al* [599], Hwang *et al* [600], and Jain and Rodriguez [154].

4.2.2 Multi-time step methods

In multi-time step methods, each local part of a flexible body is integrated in time using its own time step, thus eliminating the need to integrate the entire FMS using the smallest system time step. Small or stiff components can be integrated with small time steps while large or compliant components can be integrated using larger time steps. This can lead to considerable gains in computational speed for practical FMS, which usually involve components with disparate time scales. Multi-time step methods have not yet been used in FMD, however they have been successfully applied to large-scale nonlinear structural dynamics applications such as crash-worthiness analysis [705]. Also, they are implemented in commercial nonlinear structural dynamics explicit codes that can also be used to model FMS such as DYNA-3D and DYTRAN.

Multi-time step methods can be implemented with implicit [706] and explicit [489,705] methods. They can also be used to mix implicit and explicit integration in the same solution [488,489,706,707]. By alleviating the time step restriction of explicit solution procedures, multi-time step methods make explicit procedures competitive with implicit procedures for problems with a small number of DOFs ($\sim$1000 DOFs). Thus, multi-time step methods are mostly used in practice with explicit solution procedures.

The first multi-time step algorithms allowed only integer time step ratios [706,707] (ie, a minimum time step Δt was selected and all other time steps can only take on values of $n\Delta t$, where n is a positive integer). This restriction was relaxed for structural dynamics problems in Neal and Belytschko [705]. Two types of time step partitions can be used: nodal partitions and element partitions.

Although the area of FMD probably has a lot to gain, in terms of increasing the computational efficiency, from general multi-time step iterative-implicit and explicit solution procedures, which include an algorithm for modeling general constraints, such procedures have not yet been presented in the literature.

4.2.3 Parallel computational strategies

The development of solution procedures that can be implemented on parallel computer architectures is very important for practical FMD applications. Using a large number of processors, it may be possible to achieve real-time simulation of large-scale practical FMS. This can be used in applications such as real-time control of FMS, real-time virtual reality simulation of FMS, and computational steering. The most important aspect of a parallel solution procedure is the speedup versus the number of processors. Algorithms that achieve a linear speedup have the largest potential benefit.

Explicit solution procedures with a lumped mass matrix are embarrassingly parallel at both the element and nodal level within a time step, and have a theoretical linear parallel speedup ratio [91,699]. This means that the element forces and nodal accelerations are independent within a time step. On the other hand, implicit solution procedures, which involve matrix decompositions, cannot be easily parallelized and usually cannot achieve a theoretical linear speedup at the element level because the matrix decomposition involves interdependent operations. Implicit solution procedures based on the floating frame and absolute coordinates can be parallelized at the body level [708]. Implicit solution procedures based on the floating frame, relative coordinates, and a recursive solution procedure are difficult to parallelize at the body level because all the operations from the tip to base bodies and vice versa have to be performed in order. These algorithms can be parallelized for each branch of bodies [599,600,709] or for the evaluation of the various variables [151,710,711].

4.2.4 Object-oriented strategies

The main advantage of an object-oriented strategy is that it provides the best known mix of modularity and reusability. FMS can be naturally described using an object-oriented strategy [712]. This is because an FMS consists of modular components or objects that can be connected together in an arbitrary arrangement. The following classes of objects have been identified in the literature [709,713–720]: system components, prescribed motion, contact/impact surfaces, joints, forces, sensors, physical materials, and material colors. A detailed parametric solid geometric model of each component can be included as part of the component's data structure. Typical objects used in each of these classes are shown in Fig. 4. Each class has a set of standard properties and methods that are inherited by objects in that class. The inheritance construct allows new object types to be easily created. Communication between objects is performed only through the standard methods and properties. Object representation completely hides or *encapsulates* the underlying mathematical models. The object-oriented strategy also allows complex objects to be assembled from simpler objects. Object-oriented strategies were applied to the construction and analysis of rigid multibody systems [715,721,722] and FMS [718,720,723].

A major advantage of an effective and comprehensive object-oriented representation of FMS is that it can be used to generate many types of models wich are used in the analysis, design, and manufacturing of FMS such as finite element models, geometric solid models, machining codes, rapid prototyping coordinates, etc.

4.2.5 Computerized symbolic manipulation

Symbolic manipulation can be used to speed up the solution procedure. This is because some terms in the final equations can be factored out or canceled out in some situations. Thus, if the symbolic expression of the output can be obtained and then simplified, the number of arithmetic operations needed to obtain an output can be considerably reduced. Typically, in rigid multibody systems, a reduction in the number of arithmetic operations by a factor of five can be achieved using the symbolically simplified final expressions [724]. The manipulation and simplification of the symbolic expressions is done using a symbolic processor. Generally, the final symbolic equations are integrated numerically in time because the resulting differential equations are nonlinear and, therefore, it is very difficult to obtain closed form expressions.

Symbolic manipulation has been extensively developed and used in rigid multibody systems [725], but has only been recently applied to FMS (see Table 3). Cetinkunt and Book [323] applied computerized symbolic manipulation to flexible open-loop type flexible manipulators. By using cut-joint constraints to model closed loops, Fisette *et al* [159,324] and Melzer [328] used computerized symbolic manipulation for modeling beam type FMS. A recursive relative coordinate formulation was used to derive, symbolically, the equations of motion. Fisette *et al* [159] and Valembois *et al* [726] used power series monomials to approximate the beam shape; while Oliviers *et al* [329] used a polynomial Taylor series expansion. Shi and McPhee [330,331] used linear graphs in which nodes represent reference frames on rigid and flexible bodies, and edges represent components that connect these frames to generate the equations of motion of FMS in symbolic form. The application of the technique to spatial Euler-Bernoulli beams was presented in Shi *et al* [267,332]. Taylor, Chebyshev, or Legendre polynomials were used to approximate the beam shape.

4.2.6 Adaptive approximation strategies

During the simulation of an FMS, some part of the system may deform beyond the range of accuracy of the underlying discretization. This routinely occurs in vehicle crash-worthiness analysis, but may also occur in highly flexible multibody systems. If the simulation is started with the finest possible discretization, then the solution may be too expensive because of the small time step needed and the large number of elements. Alternatively, the simulation can start with the coarsest possible discretization provided that an algorithm for adaptively increasing and decreasing the discretization as needed is used.

Three types of adaptive strategies are currently used: h-adaptivity [490,727]), p-adaptivity [727], and modal adaptivity [106,180,728]. In h-adaptivity, the finite element mesh is refined (fission) and unrefined (fusion) depending on the level of straining which occurs during the simulation. h-adaptivity is routinely used in the area of crash-worthiness analysis. It has been applied to FMS in Metaxas and Koh [173] and Ma and Perkins [729]. The latter used it in studying the dynamics of tracked vehicles for accurately accounting for the finite length of the track segments when an Eulerian formulation is used for modeling the track. In p-adaptivity, the degree of the polynomial shape function approximation is increased or decreased depending on the amount of deformation of the element. Modal adaptivity is used in conjunction with the floating frame approach. In modal adaptivity, the number of modes used to approximate the shape of body is increased or decreased during the simulation depending on the applied forces and the angular velocity magnitude [106,180]. The number of modes can also be increased following an impact or a sudden change in kinematic structure [728].

4.2.7 Accounting for uncertainties

There are two main sources of uncertainty in modeling physical systems: assumptions and approximations in the model; and imprecision in determining the values of the system's parameters. This means that the system response cannot be determined precisely and we can only determine the bounds on the response that correspond to the known bounds

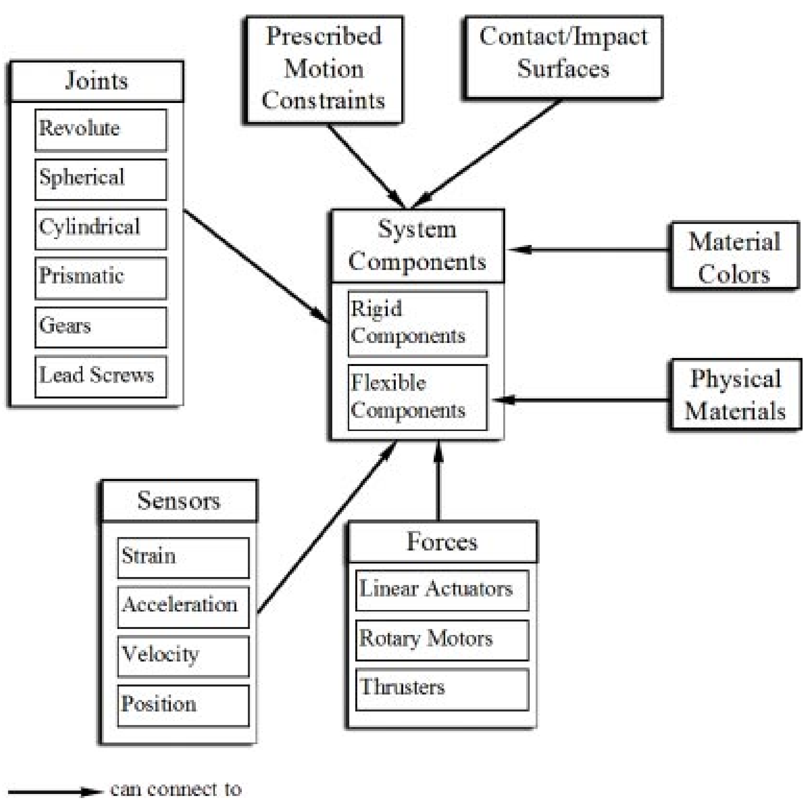

Fig. 4 Object classes

on the system parameters. Depending on the type of uncertainties present, there are three methods for assessing the effects of uncertainties on the response [730]: probabilistic methods, anti-optimization methods (or convex methods), and methods based on fuzzy set theory. If the probability distributions of the system parameters can be obtained, then probabilistic analysis is appropriate. The response in this case is obtained in terms of a probability distribution in time, which can, in general, be calculated using Monte-Carlo type simulations. When the information about the system is fragmentary (eg, only upper and lower bounds on the system characteristics are known), then anti-optimization methods can be used to find the *least favorable response* [731]. If the uncertainty is due to vague and imprecise system characteristics and insufficient information, then fuzzy-set based treatment is appropriate. The latter type of uncertainty is more prevalent in FMS because of our limited measurement technology and knowledge, and the complexity of these systems. In fuzzy-set analysis, some of the system's parameters are expressed in terms of fuzzy numbers. A fuzzy number does not have a precise value but rather can take on a range of values with each value assigned a possibility value between 0 and 1. In Wasfy and Noor [528,732,733], and Leamy *et al* [555], an approximate fuzzy-set method called the *vertex method* was used to obtain the time envelopes of the possibility distributions of various FMS response quantities given the fact that some of the system's parameters (joint characteristics, material properties, and external forces) were expressed in terms of fuzzy numbers.

5 CONTROL OF FLEXIBLE MULTIBODY SYSTEMS

The area of control of FMS is currently a very active research area due to its applications in flexible robotic manipulators [734] and articulated space structures [734–736]. Table 10 lists representative papers on control of FMS for each of these two applications. Control of FMS is concerned with finding actuator forces that produce a desired motion of the multibody system. Thus, inverse dynamics is part of control. However, control can be directly done on the physical system without a using a numerical model. This is done by using a control law along with sensors (eg, encoders, accelerometers, and strain gauges) that measure the current configuration of the system. The measurements are fed to the control law, which calculates actuator forces necessary to make the difference between the measured configuration and the desired configuration go to zero. This is called *closed-loop* control. Control can also be done in an open-loop fashion where only the initial configuration of the system is known and a force profile is *fed-forward* to the actuator to produce the desired motion. However, closed-loop control is almost always used in practical applications to be able to respond to un-modeled dynamics, disturbances, and payload variations. These effects will unavoidably make the open-loop controller diverge with time from the desired trajectory.

Three main difficulties make the control of FMS much harder than the control of rigid systems:

- *The number of DOFs is much larger than the number of actuators*. A flexible body has an infinite number of DOFs. In practice, the body can be discretized into a finite number of DOFs using a variety of techniques such as the finite element method and modal analysis. However, the number of actuators is still generally much less than the number of DOFs, which unavoidably makes the controller incapable of exactly following a desired trajectory. At best, the controller can follow a trajectory that minimizes the error between the desired and the actual trajectories.
- *Wave propagation delays*. An actuator action at one tip of a flexible link takes time to propagate to the other tip.
- *Reversed initial action*. This effect can be observed in a rotating flexible link. When a torque is applied to the link in one direction, its tip position initially moves in the opposite direction.

The last two difficulties are a result of the fact that the actuators and control points are non-collocated [737]. For example, in robotic manipulators the actuators are located at the joints and the desired position is the tip of the end-effector. Park and Asada [738] used a force transmission mechanism to reduce the distance between the control forces and the controlled endpoint, thus reducing the non-collocation between the actuator and the control point. This was shown to reduce the endpoint vibrations for a single flexible link.

FMD including forward and inverse dynamics are extensively used in the analysis and design of controllers of FMS. Forward dynamics is used in control in the following two ways:

- *Simulating the behavior of the controller*. The controller can be first tested on the numerical model to insure that the controller does not cause any type of failure (such as instability, excessive vibrations, large stresses, etc) to the physical FMS.
- *Design optimization of the controller*. Forward dynamics is used in a design optimization procedure to find the best controller parameters that meet the performance requirements (such as high maneuvering speed and small residual vibrations). The design optimization procedure typically starts by simulating the response of the system with a few sets of controller parameters. These simulations are then used to assess how changes in the parameters affect the performance. Then, the parameters are modified in such a way as to obtain a better performance. The procedure is repeated until the best performance is obtained. The design optimization procedure can also be used to find the best geometric and material parameters for the integrated FMS/controller (eg, [739]).

Similarly inverse dynamics can be used in control in the following ways:

- *Assessing the performance of closed-loop controllers*. Since, the actuator forces obtained using inverse dynamics are by definition the forces that give the closest possible

Table 10. Classification of a partial list of references on FMS control

Robotic Manipulators	**Planar/Spatial**	**Planar** Book *et al* [746], Berbyuk and Demidyuk [747], Cannon and Schmitz [748], Goldenberg and Rakhsha [749], Chalhoub and Ulsoy [639,750], Bayo [751,752], Bayo and Moulin [753], Bayo *et al* [754], Nicosia *et al* [755], De Luca *et al* [756], Sasiadek and Srinvasan [757], Yuan *et al* [758,759], Asada *et al* [745], Castelazo and Lee [760], Shamsa and Flashmer [761], Chen and Menq [762], Chedmail *et al* [420], Feliu *et al* [314], Chang [419], Aoustin and Chevallerau [763], Kubica and Wang [764], Eisler *et al* [765], Xia and Menq [766],Levis and Vandergrift [767], Ledesma and Bayo [740], Book [734], Kwon and Book [768], Yigit [769], Gordaninejad and Vaidyaraman [356], Park and Asada [738], Rai and Asada [739], Hu and Ulsoy [770], Meirovitch and Lim [771], Choi *et al* [772,773], Chiu and Cetinkunt [774], Lammerts *et al* [775], Gawronski *et al* [776], Meirovitch and Chen [777], Milford and Asokanthan [778], Yang *et al* [779], Aoustin and Formalsky [780], Mordfin and Tadikonda [781], Mimmi and Pennacchi [782]. **Spatial** Book [783], Pfeiffer [784], Ledesma and Bayo [741], Jiang *et al* [785], Ghazavi and Gordaninejad [786].
	Number of links	**Single-Link** Cannon and Schmitz [748], Goldenberg and Rakhsha [749], Bayo [751], Sasiadek and Srinvasan [757], Yuan *et al* [758], De Luca *et al* [756], Nicosia *et al* [755], Chen and Menq [762], Castelazo and Lee [760], Shamsa and Flashmer [761], Feliu *et al* [314], Chang [419], Kubica and Wang [764], Levis and Vandergrift [767], Kwon and Book [768], Park and Asada [738], Rai and Asada [739], Choi *et al* [773], Chiu and Cetinkunt [774], Milford and Asokanthan [778], Aoustin and Formalsky [780], Marghitu *et al* [687], Mordfin and Tadikonda [781], Mimmi and Pennacchi [782]. **Multi-link** Book *et al* [746], Book [783], Berbyuk and Demidyuk [747], Chalhoub and Ulsoy [639,750], Pfeiffer [784], Baruh and Tadikonda [787], Asada *et al* [745], Jonker [559], Chedmail *et al* [420], Cetinkunt and Wen-Lung [788], Aoustin and Chevallerau [763], Yuan *et al* [759], Xia and Menq [766], Eisler *et al* [765], Ledesma and Bayo [740,741], Yigit [769], Gordaninejad and Vaidyaraman [356], Hu and Ulsoy [770], Meirovitch and Lim [771], Jiang *et al* [785], Meirovitch and Chen [777], Zuo *et al* [789], Lammerts *et al* [775], Gawronski *et al* [776], Ghazavi and Gordaninejad [786], Ge *et al* [790], Ghanekar *et al* [791], Yang *et al* [779], Banerjee and Singhose [792], Xu *et al* [793].
	Control type	**Regulator control** Sasiadek and Srinvasan [757], Castelazo and Lee [760], Shamsa and Flashmer [761], De Luca and Siciliano [794], Aoustin and Formalsky [780]. **Tracking control** Book *et al* [746], Goldenberg and Rakhsha [749], Chalhoub and Ulsoy [639,750], Bayo [751], Pfeiffer [784], Yuan *et al* [758], De Luca *et al* [756], Nicosia *et al* [755], Asada *et al* [745], Chedmail *et al* [420], Chang [419], Xia and Menq [766], Ledesma and Bayo [740,741], Kwon and Book [768], Yigit [769], Gordaninejad and Vaidyaraman [356], Hu and Ulsoy [770], Meirovitch and Lim [771], Zuo *et al* [789], Lammerts *et al* [775], Gawronski *et al* [776], Chiu and Cetinkunt [774], Meirovitch and Chen [777], Ghazavi and Gordaninejad [786], Yim and Singh [795], Milford and Asokanthan [778], Yang *et al* [779], Banerjee and Singhose [792]. **Vibration control** Ider [796]. **Force control** Hu and Ulsoy [770], Yim and Singh [795].
	Feedback	**Linear state (actuator/joint) feedback Angular position (encoders)** Most references, *eg*, Milford and Asokanthan [778], Aoustin and Formalsky [780]. **Angular velocity (Tachometers)** Aoustin and Formalsky [780]. **Endpoint feedback Position** Cannon and Schmitz [748], Feliu *et al* [314], Jiang *et al* [785]. **Acceleration (Accelerometer)** Chalhoub and Ulsoy [750], Milford and Asokanthan [778]. **Force (Force sensor)** Hu and Ulsoy [770].
	Joint type	**Revolute joints** **Most references.** **Prismatic joints** Gordaninejad and Vaidyaraman [356], Hu and Ulsoy [193]. **Lead-Screws** Chalhoub and Ulsoy [639,750].
	Material model	**Linear Isotropic** **Most references.** **Composite materials** Gordaninejad and Vaidyaraman [356], Ghazavi and Gordaninejad [786].
Space Structures	**Planar/Spatial**	**Planar** Schafer and Holzach [797], Yen [798], Banerjee [482], Yen [799]. **Spatial** Krishma and Bainum [800], Banerjee [482].
	Control Type	**Retargeting flexible antennas and panels** Ho and Herber [406], Meirovitch and Quinn [409], Meirovitch and Kwak [370,801], Kakad [412], Bennett *et al* [802], Banerjee [482], Kelkar *et al* [803,804], Yen [798], Singhose *et al* [805]. **Vibration Control** Schafer and Holzach [797], Krishma and Bainum [800], Meirovitch and Quinn [409], Fisher [806], Li and Bainum [807], Banerjee [482], Su *et al* [808], Kelkar *et al* [803,804], Kelkar and Joshi [809], Dignath and Schiehlen [556].

Table 10. *(continued).*

		Attitude Control in the presence of disturbances Ho and Herber [406], Fisher [806], Ramakrishnan *et al* [242], Maund *et al* [810], Bennett *et al* [802], Cooper *et al* [811], Yen [798,799], Mosier *et al* [812], Wasfy and Noor [528], Nagata *et al* [162]. **Deployment Control** Wasfy and Noor [528].
	Feedback	**Relative displacement** Schafer and Holzach [797].
Mechanisms		**Planar Crank-slider** Liao and Sung [813], Gofron and Shabana [360,361], Choi *et al* [772], Liao *et al* [814]. **Tracking** Gofron and Shabana [360,361]. **Vibration control** Liao and Sung [813], Choi *et al* [772], Liao *et al* [814].
Type of Frame	**Floating frame**	**Most references.**
	Corotational frame	Eisler *et al* [765].
	Inertial frame	Wasfy and Noor [528].
Control law	**PID control**	Cannon and Schmitz [748], Berbyuk and Demidyuk [747], Schafer and Holzach [797], Goldenberg and Rakhsha [749], Chalhoub and Ulsoy [639,750], Pfeiffer [784], Shamsa and Flashmer [761], Chang [419], Yuan *et al* [759], Yigit [769], Gordaninejad and Vaidyaraman [356], Park and Asada [738], Tu *et al* [815], Choi *et al* [773], Ghazavi and Gordaninejad [786], Ghanekar *et al* [791], Aoustin and Formalsky [780], Wasfy and Noor [528], Mordfin and Tadikonda [781]. **Proportional** Book *et al* [746], Book [783], Gawronski *et al* [776]. **Non-linear** Castelazo and Lee [760].
	Adaptive control	Sasiadek and Srinvasan [757], Yuan *et al* [758], Chen and Menq [762], Bennett *et al* [802], Lammerts *et al* [775], Milford and Asokanthan [778], Yang *et al* [779,816].
	Robust control	Hu and Ulsoy [770], Liao *et al* [814].
	Neural-Network	Maund *et al* [810], Chiu and Cetinkunt [774], Yen [798,799].
	Pseudo-Linearization	Nicosia *et al* [755], Levis and Vandergrift [767], Nagata *et al* [162].
	Linear quadratic regulator (LQR)	Cannon and Schmitz [748], Meirovitch and Kwak [370], Chedmail *et al* [420], Feliu *et al* [314], Liao and Sung [813], Meirovitch and Lim [771], Choi *et al* [772], Su *et al* [808], Dignath and Schiehlen [556].
	Fuzzy control	Kubica and Wang [764], Zeinoum and Khorrami [817], Xu *et al* [793].
	Computed-torque method	**Reference system: Rigid-body model** Goldenberg and Rakhsha [749], Pfeiffer [784], Chedmail *et al* [420], Chang [419], Gofron and Shabana [360], Tu *et al* [815], Meirovitch and Chen [777]. **Reference system: Linearized Flexible-body model** Bayo [751,752], Bayo *et al* [754,818], Bayo and Moulin [753], De Luca *et al* [756], Asada *et al* [745], Feliu *et al* [314], Williams and Turcic [819], Kokkinis and Sahrajan [742], Gawronski *et al* [776]. **Reference system: Flexible-body model with rigid/flexible coupling terms** Pham *et al* [820], Ledesma and Bayo [740], Ledesma and Bayo [741], Gofron and Shabana [360,361], Chen *et al* [743], Xia and Menq [766], Gordaninejad and Vaidyaraman [356], Kwon and Book [768], Ghazavi and Gordaninejad [786], Lammerts *et al* [775]. **Reference system: Flexible-body model with rigid/flexible coupling terms and geometric nonlinearity** Eisler *et al* [765], Rubinstein *et al* [744], Banerjee and Singhose [792].

trajectory to the desired trajectory, a good measure of performance of the closed-loop controller is the difference between the controller's forces and the inverse dynamics forces.

- *Feed-forward open-loop control of FMS*. Inverse dynamics can be used to calculate, in advance, the actuator forces necessary to move the FMS from the initial position to a desired position. These forces can then be applied to the system. This type of control is called *computed torque method*. The computed torque method is usually used in conjunction with a secondary closed-loop controller that fine-tunes the pre-calculated torques to minimize the tracking errors and vibrations.
- *On-line real-time closed-loop control of FMS*. In this case inverse dynamics is used as the control law. In theory, this would provide the optimum control forces. However, this requires that the inverse dynamics computation be completed faster than real-time, which is currently difficult for practical FMS.

The inverse dynamics problem can, in general, be solved by using a Newton type iterative procedure on the forward dynamics solution [740–744]. It was obtained in Korayem *et al* [248] using a symbolic manipulator and the assumed mode method. Since, for FMS, the number of forces is always less than the number of response DOFs, inverse dynamics generally cannot generate the precise desired trajectory and can only achieve the closest possible trajectory to the desired trajectory. For stiff manipulators with linearized equations of motion, the inverse dynamics solution can be obtained by solving first the inverse kinematic problem and then solving the dynamic algebraic equations of motion for the system torques [745].

In Table 10, papers that deal with the control of FMS are classified according to the type of application, the deformation reference frame, and the strategy for the control law. In the Subsection 5.1, we will discuss the two main applications of control of FMS, namely, control of flexible manipulators and control of flexible space structures. In Subsection 5.2, the various types of control laws, which were applied to FMS, are reviewed. An integral part of a control system is comprised of the actuators and sensors. Brief overviews of the various actuator and sensor types and computational models used in conjunction with control of FMS are given in Subsection 5.3.

5.1 FMS control applications

Robot control is a very large research area with many dedicated journals and conferences. About two decades ago, researchers started extending their control strategies and models from rigid manipulators to flexible manipulators [79,80,746]. The direct way for extending rigid body models to flexible bodies was to use the floating frame approach. Thus, the majority of the flexible manipulators control strategies use the floating frame approach. The research on control of flexible manipulators is classified in Table 10 according to the number of spatial coordinates (planar motion or spatial motion), the number of links (one link or multiple links), control type (regulator or tracking), type of feedback, joint types, and material model. The majority of the papers presented numerical and experimental results for planar manipulators. We note that for spatial manipulators, the nonlinear centrifugal and Coriolis inertia forces take on a much more complicated form than for planar manipulators. The type of feedback is also critical for flexible manipulators. For rigid manipulators, linear state feedback, which is obtained using encoders on each robot joint, is sufficient to determine the position of the end-effector. For flexible manipulators, other types of sensors such as strain gages, accelerometers, and cameras are used to feed back to the controller the state of deformation of the manipulator.

Similarly, control of articulated space structures is a very active research area because of the need to control the shape and attitude of these structures. The following types of control operations are performed on space structures:

- *Retargeting of flexible appendages* such as antennas, solar panels, mirrors, and lens to constantly point towards a desired object. Depending on the speed of relative motion of the object, this can either be a regulator or a tracking problem.
- *Active vibration control*. Following a disturbance on the space structure such as an impact (eg, docking or mass capture) or a motion of an appendage, structural vibrations occur. These vibrations must be damped out quickly because they reduce the precision of onboard instruments.
- *Attitude control*. The orientation of the entire space structure should be controlled at all time to maintain the desired orientation. Disturbances are typically caused by the motion of an onboard appendage, the docking or separation of another structure, or solar radiation pressure. Attitude control can be achieved using control moment gyros or reaction control jets. The current orientation of the space structure can be obtained either by referring to a fixed earth target, fixed stars, or by using high-speed gimballed inertial navigation gyros [812].
- *Deployment control*. Many new space structures are deployable. They are folded in order to fit in the shroud of the launch vehicle. Then, once in orbit, they are deployed into their final configuration using mechanical joints/ actuators or inflation. In Wasfy and Noor [528], the deployment process of the Next Generation Space Telescope (NGST) was simulated. The NGST structure is deployed using revolute and prismatic joints along with rotary and linear actuators and PD controllers. Another type of deployment is deployment of space tethers, which can be used for raising/lowering the orbit of satellites and generation of electricity [555,821].

Table 10 lists the papers dealing with each of the above operations. Most references used the floating frame approach for modeling the flexible bodies. This is due to the fact that the angular velocities and accelerations for space structures are small and that these structures are usually analyzed using modal techniques. The choice of reduced modes and its effects on the controller design were discussed in Hablani [233,235,236] and Mordfin and Tadikonda [781].

5.2 Control laws

The two main requirements for an FMS controller is that it must be fast and must accurately follow the desired trajectory. These two requirements are, in general, contradictory, ie, the faster the controller the less accurate it is and vice versa. There are many types of control laws with each offering benefits under some conditions. Often, more than one type of control law is used in the same system in order to maximize the benefits. Table 10 lists the most popular types of control laws along with the papers in which they are developed and used. Control laws can be roughly divided into two main types: non-model-based laws and model-based laws (where a computer model of the FMS is used as an integral part of the control law). The non-model-based laws are:

- *Proportional-integral-derivative (PID) control*. PID control is the most widely used control law in practice. There are many situations in FMS where PID control with constant gains is not appropriate. This includes articulated multi-link FMS such as robotic manipulators because of the large configuration changes when the manipulators move and the change in centrifugal stiffening and inertia loads with the angular velocity.
- *Fuzzy control*. In fuzzy control, the controlled variables space is partitioned into overlapping ranges. A stable controller is assigned with a fuzzy membership function to each range. Then, based on the current state of the system, the desired state, and the membership function and range of each controller, a fuzzy output is calculated. This output

Table 11. Classification of a partial list of references on coupled actuator-FMS models

Electrical	**Piezo-electric actuators**	Liao and Sung [813], Zeinoum and Khorrami [817], Choi *et al* [772], Thompson and Tao [822], Yen [799], Liao *et al* [814], Maiber *et al* [823], Rose and Sachau [362], Ghiringhelli *et al* [544]. **Electro-rheological fluid actuators:** Choi *et al* [773].
Chemical	**Rocket thrusters**	**Reaction jets for space structures:** Cooper *et al* [811].
	Flywheels	**Control Moment Gyros for space structures attitude control:** Cooper *et al* [811], Wasfy and Noor [91].
Mechanical	**Pressure**	**Hydraulic actuators:** Cardona and Geradin [824], Chang [419].

is then defuzzified to yield crisp actuator forces. This strategy was applied for position and vibration control of flexible link maipulators [764,817].

- *Neural-Networks (NN)*. In this type of control, an artificial NN is trained to apply the actuation forces given the current system state and the error between the current and desired positions. This is achieved by using another controller as the training controller. The disadvantage of NN controllers is that they need to be trained using a representative variety of all possible system configurations and control scenarios. For multiple body spatial systems, this can translate into a very large training set. Chiu and Cetinkunt [774] used NN for regulation control of a single flexible link. Yen [799] proposed using NN control along with distributed piezo-ceramic sensors and actuators for tracking a desired trajectory of a flexible structure with minimum vibrations.

The model-based laws are:

- *Adaptive control*. In adaptive control, a PID type controller with adaptive gains is used. The gains are automatically adjusted during operation based on the response of the system in such a way that the response of the system closely matches that of a reference model. The forward dynamics simulation of the reference model is carried out in real time during the operation of the FMS. The difference between the response of the reference model and that of the physical system is used to adapt the PID gains and/or the reference model parameters. Since the forward dynamics problem must be solved in real time, a floating frame based reduced order modal model is often used as the reference model. One to three modes are used for each body.
- *Robust control*. In robust control, an upper and lower bound is established on the system parameters. The controller is designed to yield a stable bounded response given the range of uncertainty in the input parameters. Robust control is used in conjunction with another type of control law such as adaptive, PID control, or sliding mode control. Hu and Ulsoy [770] used the robust control strategy along with an adaptive controller for position and force tracking of a single flexible link.
- *Pseudo-Linearization* (or Feed-Back Linearization). In the pseudo-linearization method, a state/control space coordinate system is found such that the FMS in the new coordinate system has a linearized model (Nicosia *et al* [755]). A standard PID controller can then be applied in that linearized configuration. Nicosia *et al* [755] used this strategy for position tracking control of a single flexible link.
- *Linear quadratic regulator (LQR)*. In LQR control, a proportional variable gain controller is used. The gain is evaluated using a quadratic performance measure that includes the square of the difference between the actual system and a linearized model. This strategy was used for tracking control of a two-link planar manipulator in Chedmail *et al* [420], orientation regulation of a flexible link mounted on a free rigid platform in Meirovitch and Kwak [370], and tracking control of a three-link manipulator mounted on a free rigid platform in Meirovitch and Lim [771].
- *Computed torque method (CTM)*. In the CTM, the inverse dynamic torques are first obtained. These torques are fed forward to the system in an open-loop fashion. Then, another type of feedback closed-loop controller such as PID controller [776], LQR method [777], or adaptive controller [775,816] is used to fine tune the pre-calculated torques in order to minimize the tracking errors and vibrations.

A very important step in the design of a control law is to prove the stability of the controlled system. Classical linear proofs cannot be used because FMS are inherently nonlinear. Stability proofs can be done using the Lyapunov function, which measures the total energy of the system. The necessary condition for stability is that this function is strictly decreasing for an arbitrary configuration of the system.

5.3 Actuators and sensors

5.3.1 Actuators

Actuators are an essential part of a control system because they produce the forces necessary to move the FMS. Actuators convert a form of energy such as electrical, chemical, or mechanical into mechanical energy that produces forces or moments on the FMS (see Table 11 for a partial list of papers where the actuator models are coupled with FMS models). From the modeling point of view, actuators can be classified into stiff actuators and compliant actuators. Stiff actuators can be modeled as a prescribed motion because the motion they produce is not affected by the reaction forces of the FMS. For compliant actuators, the reaction forces of the FMS affect the commanded motion of the actuator. Thus there is a *two-way* coupling between the actuator and the FMS. So, a model of the actuator must be included in the model of the FMS. A typical stiff actuator is a low speed, high power rotary electric DC motor mounted on a stiff

Table 12. Classification of a partial list of references on coupled FMD models

Thermo-mechanical	**Application**	**Space structures:** Krishma and Bainum [800,826], Modi *et al* [115]. **High speed flexible mechanical systems:** Shabana [363], Sung and Thompson [364], Wasfy [85].
	Frame type	**Floating frame:** Krishma and Bainum [800,826], Shabana [363], Sung and Thompson [364], Modi *et al* [115]. **Corotational frame:** Wasfy [85].
Electro-mechanical (Mechatronics)		**Smart structures with piezo-electric actuators:** Liao and Sung [813], Thompson and Tao [822], Choi *et al*[772,773]. **Review articles:** Loewy [838], Matsuzaki [839]. **Electro-dynamics of tethered Satellites:** Leamy *et al* [555].
Fluid-structure interaction	**External flow**	**Review article:** Done [827]. **Aeroelasticity multibody beam model:** Du *et al* [365,366]. **Fluid-structure interaction for a cylinder mounted on springs:** Nomura [831]. **Fluid-flow over free and falling airfoils:** Mittal and Tezduyar [833], Johnson and Tezduyar [832]. **Floating and submerged structures:** Casadei and Halleux [834], Conca *et al* [828], Kral and Kreuzer [840].
	Internal flow	**Coupled FMS-fluid interaction:** Ortiz *et al* [829]. **Effect of explosions on containers:** Casadei and Halleux [834]. **Liquid sloshing in moving vehicles:** Sankar *et al* [841], Rumold [830].

shaft. An electric AC high speed motor is a compliant actuator because the torque it produces is inversely proportional to the angular velocity. Future FMS will be required to run at high speeds and high accelerations, and at the same time consume less energy. Under these conditions taking into account the compliance of the actuator becomes more important for accurately modeling the system dynamics.

5.3.2 *Sensors*

Sensors measure the local or global motion of a body. The measurement is sent to the controller through the feedback loop in order to adjust the controller commands. Generally, sensors are designed such that their transfer function is linear. Also, generally, the measurement action of the sensor should have negligible effect on the motion of the system. Sensors can be classified according to the type of motion that they measure into position, velocity, acceleration, and strain energy sensors:

- Position sensors measure the relative position and/or orientation of a point on the system. They include: encoders (rotary and linear, incremental, and absolute), ranging sensors (laser and light sensors, high speed cameras [748], electromagnetic tracking, and ultrasound tracking), and gyroscopes for measuring orientation.
- Velocity sensors (tachometers) measure the relative velocity.
- Accelerometers measure the absolute acceleration. Accelerometers are mostly used to measure the vibrations of flexible structures. They can also be used to measure the position, but a double integration in time is necessary which causes drift of the calculated position in time. Thus, they are usually combined with another type of lower resolution position sensor.
- The main types of strain sensors are strain gages and piezo-electric sensors [362,772,814].

Control strategies can be classified according to the type of sensor feedback of the closed-loop controller into: linear state feedback control, endpoint feedback, and strain rate feedback.

- In linear state feedback control, the sensors are collocated with the actuators. For example, in manipulators, the actuators are located at the joints and the relative joint angles are measured using encoders. This is the most widely used type of feedback.
- In endpoint feedback, the sensors and actuators are noncollocated. The feedback measurements can be used in an active controller to damp the unwanted vibrations and to correct the error in endpoint position due to the flexibility of the FMS. This feedback can be done using accelerometers [750,759,770,778], CCD cameras [314], Laser ranging sensors, electromagnetic tracking, or ultrasound tracking.
- In strain feedback, the strain at discrete points is measured as a function of time. This information can be used to estimate the deformed shape of the structure and the endpoint location as well as to measure the structural vibrations [420,759]. Thus, this type of feedback can be used in endpoint control and active vibration control.

6 COMPUTATIONAL STRATEGIES FOR COUPLED FMD PROBLEMS

FMD is primarily concerned with predicting the time history of the mechanical response (displacement, strain, and stress fields) of an FMS. The mechanical response of the FMS can be coupled with other types of physical fields such as: thermal, electric, magnetic, and fluid velocity fields. In coupled problems, the governing equations for all the fields must be solved simultaneously. A special case of coupled field problems is when the coupling between two fields is much stronger in one direction. In this case, the primary field is calculated first, independent of the secondary field, and the secondary field is then calculated using the primary field.

New applications of FMS and the need for cheaper, lighter, and faster systems are increasing the demand to perform coupled response predictions. Some of the important types of coupled FMD problems, along with examples of their practical applications, are listed in Table 12. In this section, the literature on the computational aspects of thermo-mechanical coupling and fluid-structure coupling is reviewed.

6.1 Thermo-mechanical coupling

Temperature change produces strain in a flexible body. In addition, mechanical energy losses due to material damping and friction transform into heat energy, which increases the temperature of the body. Thus, there is a two-way coupling between the deformation and temperature fields. The coupled displacement-temperature fields can be calculated by simultaneously solving the equation of motion (momentum equations) and the energy equation. There have been considerable studies of coupled thermo-mechanical problems with small deformation and large deformation [825]; however, very little work has been done on thermo-mechanical dynamic analysis of FMS (mechanical systems undergoing large rotation). The thermal effects in a FMS include:

- Heat conduction, in the bodies and between the bodies and joints
- Thermal stresses. The constitutive relation relating the stress tensor to the temperature change must be added to the stress-strain relation.
- Heat generation due to the stress power term in the energy equation
- Heat generation due to friction in the joints and on contact surfaces
- Heat flux from or to the surroundings due to radiation and convection (heat convection may be a function of the rigid body motion)
- Heat flux due to conduction when two bodies are in contact
- The physical material properties such as Young's modulus, material damping, Poisson ratio, thermal conduction coefficient, thermal expansion coefficient, etc, are a function of temperature.

The reported work on thermo-mechanical modeling of FMS has been driven by two main applications: space structures and high speed flexible mechanical systems. Space structures in orbit are subjected to severe uneven radiation heating from the sun (the temperature gradient between the side exposed to the sun and the opposite side can reach 400°C). The thermal gradients produce high thermal stresses and deformations. In addition, the energy loss due to damping from the vibrations and motion of the structure is converted into thermal energy. It is now recognized that the deployment of future large space stations and other space structures, which carry sensitive instruments, will require a much deeper understanding and accounting of the thermo-mechanical effects [115]. The reported studies have focused on one-way coupling where only the temperature affects the deformation. Krishma and Bainum [800,826] and Modi *et al* [115] developed computational methods for modeling the deflections of free beams and plates exposed to solar radiation, where the effects of surface reflectivity and the incidence angle were taken into account. Shabana [363] studied the effect of temperature on the vibrational response of a crank-slider mechanism.

Future mechanisms and manipulators are likely to be even faster and lighter than current systems, and to be made of new materials such as composites, ceramics, and plastics. Those systems are expected to generate more heat due to material damping. Since they have poor heat conduction, they are expected to be more prone to thermal deformation due large temperature gradients. Accurate modeling of the motion of these systems requires models that can account for the two-way thermo-mechanical coupling. Wasfy [85] used a corotational frame formulation and solved the fully-coupled semi-discrete momentum and energy equations to predict the thermo-mechanical response of FMS.

6.2 Fluid-structure interaction

All earthbound FMS operate in a fluid medium, mainly air or water. For relatively low speed operation in air, the effect of the fluid flow on the structural response is negligible. However, for very high speed operation in air, and operation in liquids such as water, the effect of the viscous and inertia effects of the fluid must be taken into account. A classical way to account for those effects for flexible structures is the added mass and damping method (Done [827] and Conca *et al* [828]). This method was used to account for fluid effects for helicopter blades and airplane wings (Done [827]). In Du *et al* [365,366], a 2D quasi-steady thin airfoil theory was used to calculate the aerodynamic loads for a beam attached to a moving base. Ortiz *et al* [829] used the floating frame approach to model a flexible double-link pendulum attached to a container carrying a fluid. Potential flow with modified Raleigh damping was used to model the fluid. Rumold [830] modeled planar liquid sloshing in moving vehicles using a finite-volume multigrid code for solving the full incompressible Navier-Stokes equations coupled with a multi-rigid body code.

A detailed account of the fluid flow and the interaction at the fluid-structure interface are needed for an accurate and general solution of FMS-fluid interaction problems, such as: jet engines, rotorcraft, wing propelled aircraft, water submerged mechanical systems, and fluid flow in flexible pipes (eg, blood flow). These problems can be solved by simultaneously solving the Navier-Stokes equations for the fluid and the FMS equations of motion. New computational techniques have been developed to account for the large rigid body motion of FMS while they move in a fluid medium. These include:

- The Arbitrary Lagrangian-Eulerian (ALE) formulation. This method can be used to model the fluid flow through a moving fluid domain [831–834].
- Moving the fluid mesh along with the flexible solid components smoothly and evenly by modeling the fluid mesh as a very light and very flexible elastic solid domain tied to the solid mesh [832]

- Using overlapping CFD mesh (Chimera grids) [835]. Each body can be surrounded by its own grid. The grids from different bodies overlap due to the rigid body motion and deformation. Overlapping grids have been used in the CFD simulation involving separation of multiple rigid bodies during flight [836,837].
- Automatic re-meshing of the fluid domain if the deformation of the domain is excessive [832]
- Coupling between the fluid and structural forces at the interface by writing the dynamic equilibrium of force equations at the interface nodes [834]

7 DESIGN OF FLEXIBLE MULTIBODY SYSTEMS

In addition to the ability to predict the dynamic response of FMS, two main capabilities are needed for the design of FMS. These are design representation and design optimization.

7.1 Design representation

The aim of design representation is to find an effective strategy for storing all the required information about the system. Hierarchical object oriented FMS representation strategies have been demonstrated to be very effective (see Subsection 4.2.4). An object-oriented design representation strategy can be used in a virtual product development environment to allow the following capabilities:

- *Creation of the model* in an intuitive user-friendly graphical environment
- *Automatic generation of the different types of representations needed during the design and manufacturing processes* from a single general object-oriented representation of the FMS. The types of representations include: geometric solid models, finite element models, normal mode models, CNC machining codes, rapid prototyping models, manufacturing steps/processes, assembly steps, etc.
- *Dynamic simulation*. The FMD analysis code can be embedded in the virtual product development environment to allow building the model and predicting the dynamic response in one integrated environment.
- *Visualization of the FMS design*. This involves displaying the system's model from different views with a realistic rendering during the design process so that the user can quickly make design changes.
- *Interactive Visualization of the simulation results*. This involves displaying an animation of the system motion that is calculated using the FMD code. The user can change the parameters of the visualization such as the animation speed, the model color, graphs parameters, etc. The geometric model can be overlaid on the finite element model in order to display an animation comprising the geometric details of the system instead of the idealized beam or shell finite elements. The simulation can be visualized on single-screen desktop workstations all the way up to multi-screen stereoscopic immersive virtual reality facilities [723,842].

Graphical design environments that include some of the above capabilities have been presented in the literature (eg, [714,720,843]). Also, many commercial FMD codes currently provide the above capabilities, to some degree.

7.2 Design optimization

The aim of design optimization is to obtain the system parameters that minimize an objective function, which comprises measures of the system performance requirements and the system cost while satisfying performance constraints. Predicting the system's dynamic response is needed in the course of the design optimization process in order to evaluate the objective function and/or the constraints. Strategies for design representation and design optimization of FMS coupled with FMD modeling have been developed in the following references: Schiehlen [713], Daberkow *et al* [714], Haug [709], Daberkow and Schiehlen [717], and Hardell [844]. In Table 13 a classification of a partial list of references devoted to FMS design optimization techniques is shown. The design optimization problem can be written as:

$$\min f(\lambda_i)$$
$$\text{subject to } \; g_j(\lambda_i) \leqslant 0 \quad i=1\cdots N, \quad j=1\cdots M \tag{19}$$

where f is the objective function, λ_i is design variable number i, g_j is constraint function number j, N is the total number of design variables, and M is the total number of constraints. Typical design variables include system dimensions and material properties. Typical constraints include limits on weight, stresses, and displacements. The constraints can be combined in the objective function using either Lagrange multipliers or a penalty method. The evaluation of the objective function and/or the constraints requires a forward dynamics solution for the FMS. This makes the constraint equations a nonlinear function of the design variables. Nonlinear optimization problems can be solved numerically using one or more of the following methods: gradient descent, heuristics, expert systems, and genetic algorithms (see Table 13). Gradient descent algorithms start from an initial design state and iteratively find a local minimum design state by changing the variables in the direction of the steepest descent gradient. The main limitation of a gradient descent algorithm is that the design variables must be continuous. A popular gradient descent algorithm for mechanical systems is the sequential quadratic programming technique [845–847]. If the design variables are discontinuous, discrete, or integer type parameters (such as material type, system configuration, number of supports, etc), then more suitable optimization techniques are heuristics, expert systems, and genetic algorithms. Since most design problems involve both continuous and discontinuous type variables, a hybrid optimization procedure consisting of two or more optimization algorithms can be used. Heuristics, expert systems, and gradient descent algorithms have been used in the design of flexible planar mechanisms by Imam and Sandor [66], Thornton *et al* [848], Cleghorn *et al* [849], Zhang and Grandin [850], Hill and Midha [851], Liou and Lou [852], Liou and Liu [853,854], and Liou and Patra [855]. To the authors' knowledge, there are no reported studies on the use of genetic algorithms for the design optimization of FMS.

For gradient optimization methods, partial derivatives of the objective function, and the constraint functions with respect to the design variables are needed. This requires the

Table 13. Classification of a partial list of references on FMS design optimization

Evaluation of sensitivity coefficients	**Direct differentiation**	Imam and Sandor [66], Haug [845], Bestle and Eberhard [846], Woytowitz and Hight [856], Wasfy and Noor [91], Liu [857], Pereira *et al* [372], Dias and Pereira [858].
	Finite differences	Imam and Sandor [66], Wasfy and Noor [91], Ider and Oral [859,860].
Reference frame	**Floating**	**Rigid multibody systems:** Haug [845], Bestle and Eberhard [846]. **No coupling between rigid body and flexible body motion (KED):** Imam and Sandor [66], Thornton *et al* [848], Cleghorn *et al* [849], Zhang and Grandin [850], Liou and Lou [852], Liou and Liu [853], Yao *et al* [245], Liou and Liu [854], Liou and Patra [855]. **Coupling between rigid body and flexible body motion:** Dias and Pereira [861], Ider and Oral [859], Oral and Ider [860], Pereira *et al* [372].
	Inertial	Woytowitz and Hight [856].
Applications	**Rotating beam**	Woytowitz and Hight [856].
	Manipulators	Yao *et al* [245], Rai and Asada [739], Oral and Ider [860].
	Mechanisms	Imam and Sandor [66], Thornton *et al* [848], Cleghorn *et al* [849], Zhang and Grandin [850], Liou and Lou [852], Liou and Liu [853,854], Liou and Patra [855], Hulbert *et al* [862].
	2D crash-worthiness	Dias and Pereira [861], Pereira *et al* [372].
Optimization algorithm	**Gradient descent**	**Sequential quadratic programming:** Haug [845], Bestle and Eberhard [846], Woytowitz and Hight [856], Bestle [847], Ider and Oral [859], Oral and Ider [860], Hulbert *et al* [862]. **Feasible direction method:** Dias and Pereira [861], Pereira *et al* [372].
	Heuristics/ gradient descent	Imam and Sandor [66], Thornton *et al* [848], Cleghorn *et al* [849], Zhang and Grandin [850]. **User driven Newton-Raphson iterations:** Hill and Midha [851].
	Expert system/ heuristics	Liou and Lou [852], Liou and Liu [853,854], Liou and Patra [855].
	Genetic algorithms	No references.

evaluation of partial derivatives of the response variables with respect to the design variables. This can be done either by direct differentiation of the equations of motion or by finite differences (see Table 13). In the direct differentiation approach, if the semi-discrete equations of motion are written as:

$$M^e \ddot{x}^e = f^e_{\text{int}} + f^e_{\text{ext}} \tag{20}$$

then direct differentiation of Eq. (20) yields:

$$M^e \frac{\partial \ddot{x}^e}{\partial \lambda_j} = \frac{\partial f^e_{\text{int}}}{\partial \lambda_j} + \frac{\partial f^e_{\text{ext}}}{\partial \lambda_j} - \frac{\partial M^e}{\partial \lambda_j}\ddot{x}^e \tag{21}$$

where λ_j is design variable number j. In the direct differentiation approach, in addition to solving Eq. (20), Eq. (21) must be solved N times for the N-sensitivity coefficients $\partial x^e/\partial \lambda_j$ [91]. However, the use of the automatic differentiation facilities for generating the governing equations for the sensitivity coefficients (Eq. (21)) alleviates the complexity associated with the direct differentiation approach. However, this is accomplished at the expense of additional storage and computational time. In addition, some types of design variables involve discontinuous operators such as absolute value, maximum, or minimum operators. Examples of these variables are maximum allowable stresses and deflections. The values of these variables can shift discontinuously in both space and time. The gradients of these variables are very difficult to evaluate using the direct differentiation approach because analytical derivatives cannot be defined at discontinuities.

The finite difference approach requires $N+1$ evaluations of Eq. (20). The finite difference approach is simpler to implement since it does not involve formulating new equations and variables. In addition, gradients of discontinuous variables can readily be calculated using finite differences.

8 EXPERIMENTAL STUDIES

In the past, design and analysis of practical FMS relied primarily on experiments. Starting in the 1980s, computer speeds and the advances in computational modeling has allowed a much greater reliance on computer models. Experimental studies are, however, still very important because they can be used to develop, improve, and assess the accuracy of numerical models. In Table 14, experimental studies reported in the literature are listed and classified by application.

9 FUTURE RESEARCH DIRECTIONS

As in other fields, the future research directions of FMD will be driven by the applications. Some of the recent and future applications are outlined in Subsection 9.1. Those applications will likely require higher model fidelity and faster computational speed. Research topics that are likely to produce

Table 14. Classification of a partial list of references on FMS experimental studies

Mechanisms	**Four-Bar Linkage**	Alexander and Lawrence [863], Jandrasits and Lowen [11], Turcic *et al* [310], Thompson and Sung [352], Sung *et al* [864], Liou and Erdman [26], Sinha *et al* [865], Liou and Peng [385], Giovagnomi [866]. **Composite materials:** Thompson *et al* [351], Sung *et al* [353].
	Crank-Slider	Thompson and Sung [352], Sung *et al* [864]. **Composite materials:** Sung *et al* [353]. **Smart materials:** Choi *et al* [772], Thompson and Tao [822]. **Joint clearances:** Soong and Thompson [633].
	5 links or more	Caracciolo *et al* [867].
	Machines	Chassapis and Lowen [387].
Manipulators	**One link**	Cannon and Schmitz [748], Feliu *et al* [314], Liou and Peng [385], Kwon and Book [768], Milford and Asokanthan [778], Aoustin and Formalsky [780]. **Smart materials:** Choi *et al* [773].
	Two or more links	Chalhoub and Ulsoy [750], Pan *et al* [340], Chedmail *et al* [420], Yuan *et al* [759], Hu and Ulsoy [770], Yang *et al* [779], Lovekin *et al* [868], Gu and Piedboeuf [869,870].
Tracked Vehicles		Choi *et al* [646], Nakanishi and Isogai [430].
Space structures		Mitsugi *et al* [871], Lovekin *et al* [868].

improved model fidelity and speed are identified in Subsection 9.2. The new models must then be integrated in the design process of FMS. Research topics that can help in integrating FMD models into the design process are identified in Subsection 9.3.

9.1 Recent and future FMS applications

The current trend in the various applications of FMS is towards cheaper, lighter, faster, more reliable, and more precise systems. In addition to traditional FMS applications listed in Section 1, some of the recent applications, which will likely require more FMD research in order to improve the model fidelity and computational speed, include:

High speed, lightweight manipulators. Currently manipulators are constructed using bulky stiff links and are moved at slow speeds so that they do not experience excessive deflections and vibrations. New lightweight stiff materials, piezo-electric actuators and sensors, and high speed model-based closed-loop control are pushing the speed and weight limits of manipulators. These new manipulators can be used in a wide array of applications such as industrial production, nuclear waste retrieval, and fast assembly of space structures in orbit.

Large high precision deployable lightweight space structures. Stable and high dimensional precision space structures are needed for new high resolution and high sensitivity optical and radio telescopes as well as very high bandwidth communication satellites. Those space structures will be deployed in orbit from a small package that fits in the shroud of the launch vehicle into their large useful configuration. Effects such as joint friction, material damping, thermal heating, and solar radiation pressure must be included in those models.

High speed, lightweight mechanisms. New lightweight stiff materials such as advanced composites and ceramics are increasingly being used in automobile and airplane engines and production machines. The flexibility effects in these mechanisms will be larger than current mechanisms and more difficult to model due to material nonlinearity and anisotropy. In addition, complex material failure modes will make prediction of allowable operation limits more difficult.

Bio-dynamical systems. Typical applications include: limb replacement; vehicle occupant crash analysis; motion/force analysis for athletes, animals, and insects.

Robots. There is an increasing interest to develop intelligent autonomous robots that can perform tedious tasks instead of humans. These robots must have an effective control strategy to enable them to walk on rough terrains and manipulate, grasp, and move objects using arms and hands. Those robots are also likely to be lightweight and flexible.

Active model based control of robots, manipulators, and space structures

Micro and nano electro-mechanical systems (MEMS and NEMS). These systems have many applications in the medical, electronics, industrial, and aerospace fields; and, therefore, have been receiving increasing attention from researchers in recent years. MEMS have dimensions ranging from a few millimeters to one micrometer, while the dimensions of NEMS range from submicron dimensions down to nanometer/atomic scale. There are already practical applications of MEMS, such as airbag deployment accelerometers, and NEMS such as carbon Nanotube manipulators and probes [872]. Typically, MEMS and NEMS involve at least one moving component that is coupled with an electric and/or magnetic field. Due to their small size, viscous fluid flow effects can affect the motion. MEMS can be modeled using the classical mechanics techniques presented in this paper. For atomic sized NEMS, quantum effects are important and can be modeled using classical molecular dynamics, tight-binding molecular dynamics, or density functional theory (a theory used to solve the multibody nuclei-electrons Schrodinger equation), which are various levels of approximations for the atomic forces [873]. Many MEMS and NEMS include: components that undergo large rigid body motion while experiencing deflections and vibrations; kinematic joints; control actuators/motors; and sensors. In addition, many MEMS and NEMS such as manipulators and gears [874] experience frictional contact/impact. Therefore, many of the modeling methodologies developed for classical FMS can be adapted to MEMS and NEMS.

Coupling of physical experiments and simulations. The cost and number of physical tests of FMS, can be greatly reduced by coupling the physical experiment to the simulation [842]. For example, the physical test can be performed on an automobile suspension system while the rest of the vehicle is simulated. By using actuators and sensors at the interface between the physical test and the simulation, the interface forces required for the test and simulation can be generated.

Real-time interaction with virtual FMS. In virtual reality applications, the user interacts with a computer generated environment. The interaction can range from manipulating the virtual objects using the mouse and keyboard to touching and holding the objects using haptic gloves [842]. A real-time FMD simulator can be used to generate the both visual and haptic feedback such that the virtual objects behave like real-world objects [173,875]. Applications range from 3D computer games to training.

Movies and computer games. FMD models can be used to generate a realistic visual animation of the motion and contact/impact response of various objects.

9.2 High performance FMS models research

In order to design, construct, and operate FMS that satisfy the current and future applications requirements, more research is needed to improve FMD models fidelity and computational speed. This will reduce the reliance on physical prototypes, thereby reducing the development cost and time. Model fidelity can be improved by incorporating all the relevant phenomena affecting the response into the model. Computational speed is especially needed for inverse dynamics and design optimization problems because of the large number of iterations involved in those solution procedures. In addition, some new applications, such as model based control and interacting with FMS in virtual reality environments, require real-time or near real-time response prediction. In the past, accuracy was sacrificed in favor of computational speed because, otherwise, practical FMS problems could not be solved in a reasonable amount of time on existing computers. Currently, the increasing speed of computers provides opportunities for high-fidelity rapid simulations of complex FMS. Improving FMD model fidelity and speed requires more research in the following subtopics of FMD.

9.2.1 Basic models

More research is needed to improve the basic models of the flexible components. These include:

- *Accurate and efficient beam, shell, and solid elements*. Accuracy requires that the element does not exhibit any type of locking or spurious modes and that it must pass all accuracy tests. Efficiency means that the element is not prohibitively expensive relative to other available elements that can solve the same problem to the same accuracy. The element must accurately account for the following: large arbitrary spatial rigid body rotation, large deflections, large strains, transverse shear deformation, rotary inertia, initial curvature, twisted (warped) beams and shells, arbitrary cross sections, general nonlinear anisotropic material constitutive law including material damping and friction, and material failure.
- *Contact/impact friction models*. Traditionally, friction is modeled using a Coulomb friction model. However, more sophisticated models such as asperity based models (eg, [876,877]) exist and need to be incorporated in FMS contact/impact models. Friction is likely to be very important in applications such as docking and assembly of space structures, and grasping payloads using robotic manipulators.
- *Joint models*. More research is needed to assess velocity-force/moment relation (including friction and damping), clearances [70,637], and dimensional precision and hysteresis of joints (Wasfy and Noor [733]). These effects are not critical for low speed and/or low precision systems. However, for future systems, understanding those effects will be very critical for the design of high performance joints.

9.2.2 Formulations

An understanding of the mathematical foundations of existing formulations is needed. This includes the following:

- *Mathematical relation between the three types of reference frames*. Further research is needed to determine the mathematical relations between the three reference frame formulations for the various types of elements, model reduction methods, and mass matrix types (lumped or consistent). This can help in identifying the assumptions, the limitations, and the range of validity of the response of each formulation. Some studies have shown the equivalence of the corotational and the inertial frame formulations [453]. Also, if the flexible motion inertia forces in the floating frame approach are referred to the global frame, then the floating frame can be considered as one corotational frame for the entire body.
- *Rotational DOFs for the corotational and inertial frames*. In corotational and inertial frame formulations, many types of rotational DOFs are used such as Euler angles, incremental rotation vector, rotation pseudo vector, rotation tensor, and global slopes (see Tables 3 and 4). In some studies, rotational DOFs are not used [85,91,527]. More research is required to determine the advantages and limitations of the various types of rotational DOFs, particularly their effect on the rotational inertia moments. Also, more research is needed to determine the advantages and limitations of formulations that use rotational DOFs versus formulations that do not use them.
- *Hybrid frame formulations*. These are formulations where more than one type of reference frame is used in the same problem. This can be advantageous for FMS with disparate ranges of rotational speeds and/or relative deformations of the flexible components. Hybrid formulations will require developing solution procedures that can handle multiple

reference frames for inertia and internal forces, different types of rotational DOFs, and multiple time step sizes.

- *Effect of nonlinearities on modal coordinates*. The floating frame approach in conjunction with modal reduction is used extensively for space structures and flexible manipulators. However, guidelines should be developed for the range of angular velocities, stiffness, and system configurations within which modal coordinates produce accurate results. Also, nonlinear modal reduction methods need to be further developed in order to accurately account for nonlinearities (centrifugal stiffening, foreshortening, and large deflections) and changes in kinematic structure (addition/deletion of joints and constraints).

9.2.3 Computational strategies

Improved computational strategies are needed, which include enhancements in:

- *Solution strategies*. Guidelines are needed for choosing implicit and explicit solution procedures. Future research should address developing mixed explicit-implicit multi-time step solution procedures for FMS to maximize the advantages of both solution methods.
- *Parallel solution procedures*. Procedures that can achieve a linear speedup of the number of processors to the number of DOFs are the most advantageous. Explicit methods naturally satisfy this condition. More research is needed to develop implicit or implicit-explicit hybrid methods that achieve a near linear speedup. Also, more research is needed to implement the parallel solution procedures on new, massively parallel, heterogeneous computer clusters in such a way as to minimize the idle time of each processor and the volume of communication between processors.
- *Adaptive strategies*. Further research is needed to incorporate h, p, and modal adaptive methods to FMS. Also, further research is needed for model adaptation in which the reference frames, element formulations, etc, can be switched during the simulation.
- *Symbolic Manipulation*. Symbolic manipulation can reduce the number of mathematical operations needed during the numerical simulation. Symbolic manipulation has been recently used in conjunction with the floating frame formulations (eg, [159,324]); however, it has not been applied to the corotational or inertial frame formulations
- *Accounting for uncertainties and variabilities*. FMS have inherent uncertainties due to assumptions and approximations in the model and imprecision in estimating the values of the system's parameters. Computational procedures that can predict the response under these uncertainties and variabilities need to be developed. More research is needed to develop and apply techniques based on probability theory, fuzzy set theory [732], and interval analysis [731].

9.2.4 Coupled FMD analyses

This is perhaps the field which will experience the largest growth in the near future because it is grossly underdeveloped and, at the same time, there are many practical applications in biomechanics, aeronautics, space structures, and micro and nano-mechanical systems that require coupled analyses. Noteworthy examples include: coupling of the dynamics of electro-magnetic and piezo-electric actuators and sensors for smart structures and MEMS; thermo-mechanical coupling for space structures, and fluid-structure interaction for submerged mechanical systems.

9.2.5 Verification and validation of numerical simulations

In order to verify and validate the accuracy of the numerical simulation benchmark, experimental and numerical test problems are needed.

- *Benchmark experiments*. These are needed in order to validate and assess the accuracy of the computational models in representing key effects such as: spatial motion, open/closed loops, high speed rotation, large deflections, etc. Most past experimental studies focused on simple FMS (eg, rotating beams, two-link manipulators, four-bar linkages, and crank-sliders) that are designed to highlight only one or two of these effects (see Table 12). While these results are useful, more results that cover various orders of magnitude and combinations of these effects are needed. In addition, there is also a need for benchmark experimental results of large practical FMS. State-of-the-art sensors (see Subsection 5.3.2) and data acquisition facilities should be used in these experiments in order to provide detailed high resolution measurements of strains and displacements (eg, [869,870]).
- *Benchmark simulations*. There is a need to develop a set of benchmark simulations for verification and comparison of the computational models. Those tests must be designed to target individual effects as well as coupled effects. A subset of those accuracy tests are the beam, shell, and solid elements benchmark tests developed in the finite element structural analysis field (eg, [567,568]). In addition, FMS accuracy tests for the following effects are needed: centrifugal stiffening, high accelerations, vibrations (mode shapes and natural frequencies), frictional contact, large arbitrary rigid body motion, and very long simulation times.

9.3 FMS design research

For typical mechanical systems, the computer analysis/simulation time is now only a small fraction of the total design process time. Most of the time is spent in formulating the problem, generating the computer model, and post-processing the results. The following technologies, when integrated with FMD techniques, can significantly reduce the design time and help design better performing (ie, close to optimum) FMS:

- *Object oriented strategies*. An object-oriented strategy can effectively couple design, simulation, and manufacturing tools, which will result in large savings in product development time and cost. This is consistent with the current trend of transforming CAD systems into virtual product development systems with embedded numerical simulation tools.
- *Design optimization methods*. FMS involve continuous, discontinuous, discrete, and integer type design variables.

While there are many papers on gradient descent methods (see Table 11), these methods work only on continuous variables. There is significantly less work on knowledge-based expert systems and there is virtually no work on the use of fuzzy expert systems and genetic algorithms for the design of FMS. The latter two techniques have proved very effective for many other types of nonlinear optimization problems, thus their application to FMS is likely to be very beneficial. For example, using fuzzy expert systems in conjunction with fuzzy-set models [732], some of the system design parameters can be defined using linguistic values. The linguistic description is more natural for humans. In addition, classifying the ranges of the parameters using the linguistic quantifiers can help in exploring a large design space faster.

- *Virtual reality*. This technology can help analysts and designers to visualize, construct, and interact with FMS models on a computer. Virtual reality can be integrated with FMD in two ways. First, it can be used as a tool for constructing the FMS geometry. Second, a near real-time forward dynamics capability can be incorporated in a virtual reality environment for interacting with the FMS using hand worn gloves and other haptic devices. This offers the user a realistic visual view as well as realistic motion and reaction forces behavior of the FMS.
- *Collaborative design and analysis of FMS*. Collaborative visualization and simulation environments allow geographically dispersed teams to work together in developing and analyzing virtual prototypes of FMS. These environments will significantly reduce the development time, lower life cycle costs, and improve the quality and performance of future FMS. The Internet can provide the communication infrastructure for these environments.

ACKNOWLEDGMENTS

The present research is supported by NASA cooperative agreement NCC-1-01-014. The authors would like to thank the following individuals for providing suggestions, comments, and additional references: Jorge Ambrósio, Universidade Tecnica de Lisboa; FML Amirouche, University of Illinois at Chicago; Alberto Cardona, Universidad Nacional del Litoral; Michel Géradin, Université de Liége; Faramarz Gordaninejad, University of Nevada; Edward J Haug, University of Iowa; Ronald Huston, University of Cincinnati; Adnan Ibrahimbegovic, LMT Cachan; JB Jonker, Universiteit Twente; John McPhee, University of Waterloo; Loc Vu-Quoc, University of Florida; Werner Schiehlen, University of Stuttgart; Ahmed A Shabana, University of Illinois at Chicago; and Brian Thompson, Michigan Technological University.

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__Tamer Wasfy__ is president of Advanced Science and Automation Corp. He received a BS with honors in mechanical engineering (1989) and an MS in materials engineering (1991) from the American University in Cairo and a PhD in mechanical engineering (1994) from Columbia University. Prior to his present position, Wasfy was a research scientist at the University of Virginia, Center of Advanced Computational Technology at NASA Langley Research Center from 1995 to 1998 and a post-doctoral research fellow at Columbia University from 1994 to 1995. He authored and coauthored over 35 articles on flexible multibody dynamics, belt-drive dynamics, computational fluid dynamics, the finite element method, visualization of large-scale datasets, virtual reality and intelligent natural-language software agents. He is a member of ASME, AIAA, and SAE.

__Ahmed K Noor__ Eminent Scholar and William E Lobeck Professor of Aerospace Engineering, Old Dominion University, Director of the Center for Advanced Engineering Environments at NASA, Adjunct Professor of Mechanical and Aerospace Engineering, University of Florida, Gainesville. From 1990–2000, he was the Ferman W Perry Professor of Aerospace Structures and Applied Mechanics and the Director of the University of Virginia's Center for Advanced Computational Technology at NASA Langley. Noor received his BS degree with honors from Cairo University and his MS and PhD from the University of Illinois at Urbana-Champaign. He taught at Stanford, Cairo and George Washington universities, as well as the universities of Baghdad and New South Wales. He has edited 30 books and authored over 350 papers in the fields of advanced design and synthetic environments and learning technologies, aerospace structures, structural and computational mechanics, multiscale modeling and new computing systems. Noor is a fellow of ASME, AIAA, ASCE, AAM and USACM. He served on a number of the National Research Council/National Academy of Engineering and NSF committees. He is Editor-in-Chief of Advances in Engineering Software, Associate Editor of Applied Mechanics Reviews and serves on the Editorial Board of several international journals. He received a number of awards including the 1989 ASCE Structures and Materials Award for exceptional contributions to the advancement of aerospace technology in civil engineering, the Technical Achievement Award from the National Academy of Engineering in 1995 and the Distinguished Probabilistic Methods Educator Award of SAE International in 2000.