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Dynamics of two mutually delay-coupled semiconductor lasers

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Abstract

We investigate numerically the dynamics of two semiconductor lasers mutually coupled via their optical fields in the particular case of a small separation between them. We show that, depending on the frequency detuning and the coupling between the lasers, the system shares the main features of semiconductor lasers subject either to constant monochromatic optical injection or to optical feedback from a distant reflector.

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1. Introduction

The nonlinear dynamics of mutually delay-coupled semiconductor lasers has attracted much attention in recent years as this system is an excellent, experimentally realizable, example of coupled nonlinear oscillators where the coupling delay is not negligible in comparison to the typical timescales of the individual elements. Localized synchronization in two mutually coupled lasers with different relaxation oscillation frequencies was demonstrated [1] and spontaneous symmetry-breaking between identical laser diodes operating in a low-frequency fluctuation regime reported [2]. Two-section laser devices were shown to be good candidates for high-speed optical signal processing in [3]. The properties of stationary and periodic solutions were investigated in the long coupling delay limit [4]. A thermodynamic potential picture of the complementary effects of spontaneous emission noise and frequency detuning was derived in [5]. Symmetry-breaking in periodic solutions and high-frequency periodic oscillations were predicted in the case of identical laser diodes with a short coupling delay [6].

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In this paper, we investigate the dynamical behavior of two mutually coupled lasers with a short coupling delay in comparison to the period of the relaxation oscillations when the coupling and the frequency detuning between the lasers are varied. Depending on the parameters, the system presents features similar to those of unlocked semiconductor lasers with constant, monochromatic optical injection [7–9, and references therein] or those of semiconductor lasers with delayed optical feedback in the short cavity regime [10–12]. In addition, we report a continuous and cyclic transition between in phase and antiphase periodic oscillations of the laser outputs as the detuning frequency or the coupling phase are varied.

2. Model

The system that we consider consists of two single-mode laser diodes coupled by mutual injection of their optical fields (see Fig. 1). Internal and operating parameters are assumed to be identical for both lasers, the frequency detuning excepted. This system can be modeled by the following dimensionless rate equations [1]:

$$\frac{dE_{1,2}}{ds} = (1 + i\alpha)N_{1,2}E_{1,2} + \eta_{2,1}E_{2,1}(s - \theta) \times \exp(-i\Omega\theta) + i\delta_{1,2}E_{1,2}, \quad (1)$$

$$T\frac{dN_{1,2}}{ds} = P - N_{1,2} - (1 + 2N_{1,2})|E_{1,2}|^2, \quad (2)$$

$E_{1,2}$ and $N_{1,2}$ are the normalized slowly varying complex electric fields and the normalized excess carrier numbers in lasers 1 and 2, respectively. The dimensionless time s is measured in units of the photon lifetime τ_p: $s = t/\tau_p$. η_1 and η_2 are the normalized coupling rates between the lasers. They are assumed identical in this work. θ is the ratio of the flying time of the light between the lasers to the photon lifetime: $\theta = \tau/\tau_p$. δ_i is the normalized detuning frequency of laser i from the normalized angular frequency Ω chosen as a common reference. In the following, we will consider the frequency of laser 1 as the reference; as a result, $\delta_1 = 0$. In this reference frame, the coupling phase is a parameter that does not depend on the relative detuning $\delta \equiv \delta_2$ between the lasers. The present choice is expected to compare well with experimental conditions where the frequency of one of the lasers could be tuned through modification of its temperature, the frequency of the second laser being kept constant. α is the linewidth enhancement factor, P is the dimensionless pumping current above solitary laser threshold and T is the ratio of the carrier lifetime to the photon lifetime. We use typical values for the linewidth enhancement factor and the ratio of the carrier lifetime to the photon lifetime, namely $\alpha = 4$ and $T = 1710$. The numerical values of the normalized flying time and pumping current are $P = 1.155$ and $\theta = 20$. For a photon lifetime $\tau_p = 1.11$ ps, the value of θ we chose here corresponds to a flying time $\tau = 22.2$ ps or equivalently to an inter-laser separation of 6.66 mm. The system operates therefore in a short cavity regime since the coupling frequency $f_c = 45.05$ GHz is much larger than the relaxation oscillation frequency $f_r = 5.27$ GHz.

Inspection of Eqs. (1) and (2) reveals that their solutions remain unaffected if the phase $\Omega\theta$ is modified by a multiple of π.

Fig. 1. Schematic representation of two semiconductor lasers mutually coupled via their optical fields.

3. Numerical results

Figs. 2 and 3 summarize the dependence of the system dynamics on the coupling rate and the detuning frequency between the lasers for two particular values of the coupling phase, namely $\Omega\theta = 3$ and $\Omega\theta = 3 + \pi/2$. Figs. 2(a) and 3(a) are bifurcation maps where stationary solutions are represented in black, period-1 periodic solutions in dark gray, period-2 periodic solutions in light gray and higher periodic, quasiperiodic and chaotic solutions are in white. Each map was deduced from 200 bifurcation diagrams computed for values of δ regularly

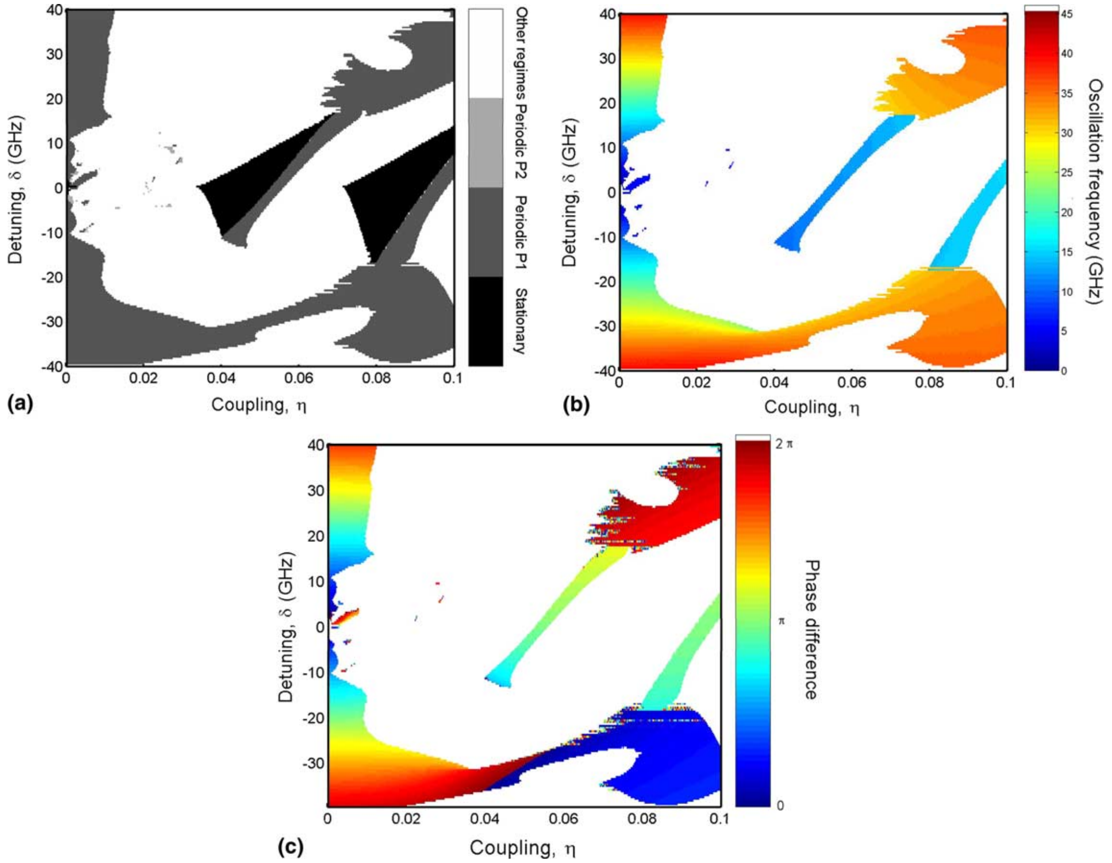

Fig. 2. (a) Bifurcation map. The coupling rate η and the detuning frequency δ are the control parameters. (b) Oscillation frequency of the period-1 periodic solutions as a function of the coupling rate and the detuning frequency. (c) Relative phase between the periodically oscillating laser intensities. $\Omega\theta = 3$.

spaced in the range [-40 GHz, 40 GHz]. Every bifurcation diagram was calculated from numerical integration of Eqs. (1) and (2) with the coupling η as the bifurcation parameter. This parameter was varied from 0 to 0.1 with a step of 2.5×10^{-4}. The nature of the solutions in the different parameter regions was detected by analyzing the successive extrema of the time series of the intensity emitted by laser 1 ($|E_1|^2$) and laser 2 ($|E_2|^2$). The bifurcation maps deduced from the outputs of laser 1 and 2 are identical because the lasers exhibit the same type of dynamics (e.g., periodic or chaotic behaviors) for a given set of parameters. Figs. 2(b) and 3(b) show the oscillation frequency of period-1 periodic solutions with respect to the coupling and the detuning. Figs. 2(c) and 3(c) depict the relative Hilbert phase between the periodically oscillating laser intensities as functions of η and δ.

Stationary solutions are observed in several disconnected regions of the parameter space interspersed with regions of more complex behaviors, such as periodic, quasiperiodic and chaotic oscillations. The bifurcation maps clearly show that the mutually coupled laser diodes do generally not follow the period-doubling route to chaos that is typical of laser diodes subject to constant, monochromatic, optical injection [9]. A tedious analysis of each bifurcation diagram reveals that the system experiences a quasiperiodic route to chaos for increasing coupling in most of the detuning range, although period-doubling can be observed

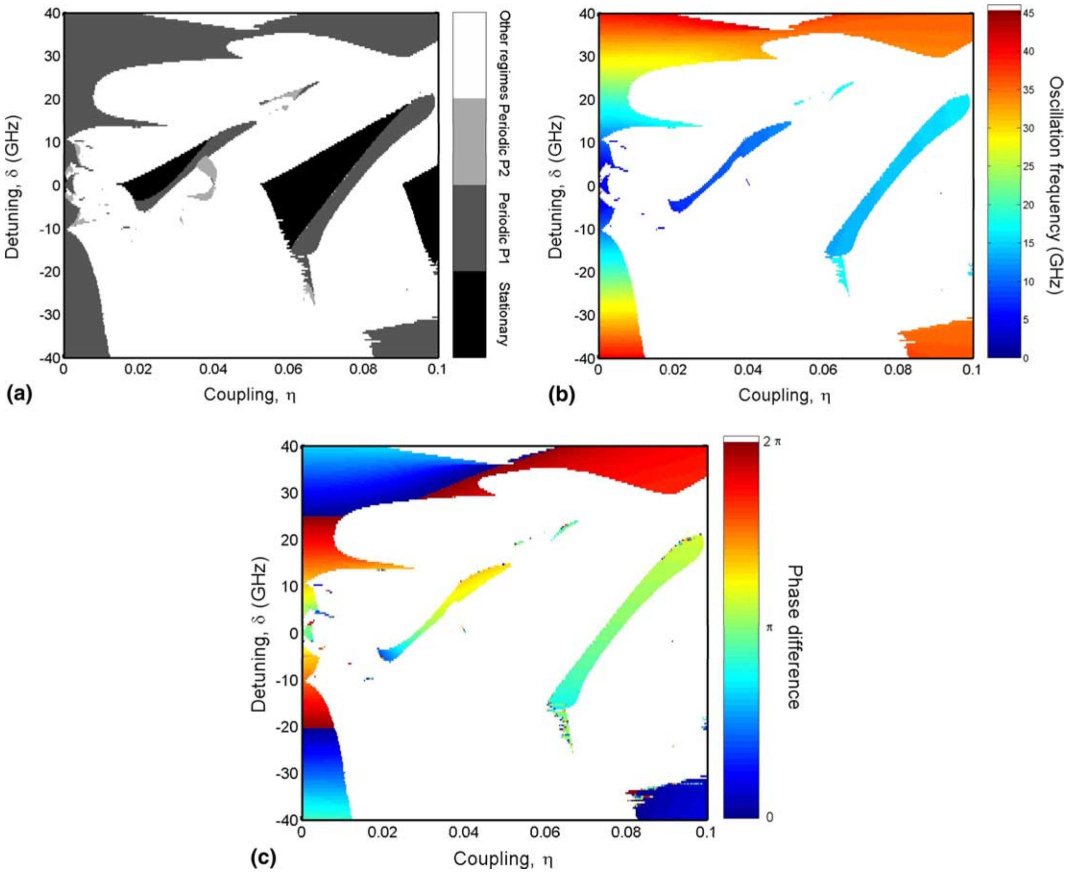

Fig. 3. Same as Fig. 2 but with $\Omega\theta = 3 + \pi/2$.

at low coupling and small detuning. By contrast, it appears that the system switches abruptly from chaotic to periodic or stationary states for increasing coupling in most of the detuning range. As shown in the following, the lasers are synchronized when their dynamics is periodic. They remain synchronized even when the system exhibits more complicated dynamics such as the emission of regular packages of pulses. In this regime, the roles of leader and laggard are determined by the sign of the detuning similarly to what was reported in the case of two mutually coupled laser diodes operating in the low-frequency fluctuation regime [2]. In the absence of detuning, these roles can switch at the end of each package of pulses.

In the stationary regions, the lasers are necessarily frequency locked. The solutions are of the form $E_{1,2}(s) = A^s{}_{1,2}\exp[\mathrm{i}(\Delta^s - \Omega)s]$ and $N_{1,2}(s) = N^s{}_{1,2}$, $A^s{}_{1,2}$, $N^s{}_{1,2}$ and Δ^s being constant. A first region of stationary solutions is found at very low detuning and coupling. It is so small that it is barely visible in Figs. 2 and 3. For increasing coupling, the stationary solutions lose their stability through Hopf bifurcations and both lasers start to oscillate periodically with a frequency very close to the relaxation oscillation frequency $f_r = 5.27$ GHz of the individual lasers in absence of coupling. The optical spectra of both lasers are almost symmetric with respect to the optical carrier frequency and exhibit peaks close to the free-running relaxation oscillation frequency and its harmonics. The amplitudes of the corresponding peaks slightly differ in the presence of detuning but are identical in its absence as shown in Figs. 4(a) and (b). In

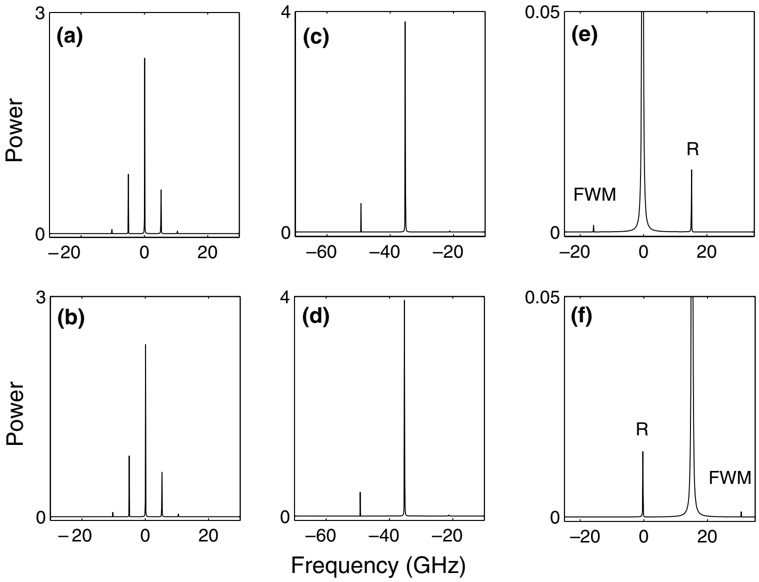

Fig. 4. (a,b) Optical spectra of laser 1 and laser 2, respectively, for $\eta = 0.004$, $\delta = 0$, $\Omega\theta = 3 + \pi/2$. (c,d) Same but with $\eta = 0.08$ and $\delta = 5$ GHz. (e,f) Same but with $\eta = 0.007$ and $\delta = 15$ GHz.

this regime, the lasers remain locked as the respective optical carrier frequencies coincide.

Other regions of stable stationary solutions are observed for higher coupling and their size increases with the coupling rate. These stationary solutions loose also their stability through Hopf bifurcations. The solutions in the adjacent periodic regions are characterized by a high oscillation frequency (>10 GHz) and are intrinsically different from those observed in the first periodic window. The optical spectra are slightly different (identical in the absence of detuning) and characterized by the existence of two main peaks located at the same frequencies for both lasers, the harmonics being barely visible (see Figs. 4(c) and (d)). As shown in [6], these periodic solutions result from a beating between two pairs of single frequency rotating wave solutions of the form $E_{1a,b}(s) = A_{a,b}$ $\exp[i(\Delta_{a,b} - \Omega)s]$ for laser 1 and $E_{2a,b}(s) = B_{a,b} \exp$ $[i(\Delta_{a,b} - \Omega)s]$ for laser 2, respectively, and the beating frequency $|\Delta_a - \Delta_b|$ increases from a periodic window to the next one and tends to an upper limit $f_\infty = 1/2\tau$ ($f_\infty = 22.5$ GHz for the flying time considered here). The frequency maps (Figs. 2(b) and 3(b)) show that the beating frequency is slightly affected by the detuning: it increases with positive detuning and decreases with negative detuning. Furthermore, the effect of the detuning decreases with the coupling rate. The lasers remain locked in these periodic windows that are adjacent to regions of stationary solutions.

In addition to the windows of injection-locked periodic solutions that we have described so far, the bifurcation maps reveal the existence of large periodic regions that are typically observed at low coupling rate and extend at higher coupling rates within limited ranges of detuning (from -40 to -20 GHz and from 20 to 40 GHz for $\Omega\theta = 3$ and from -40 to -30 GHz and from 30 to 40 GHz for $\Omega\theta = 3 + \pi/2$). Figs. 2(b) and 3(b) show that in these regions the modulation frequency of the laser outputs is almost equal to the detuning frequency for low coupling, increases with the coupling and does not exhibit a stair-like behavior as the detuning increases, contrary to what is observed in the case of mutually coupled lasers with longer coupling delay [13,14]. The corresponding optical spectra are characteristic of four-wave mixing in optically injected lasers [8] with a large peak at the laser frequencies indicating that the lasers are unlocked and two other peaks corresponding

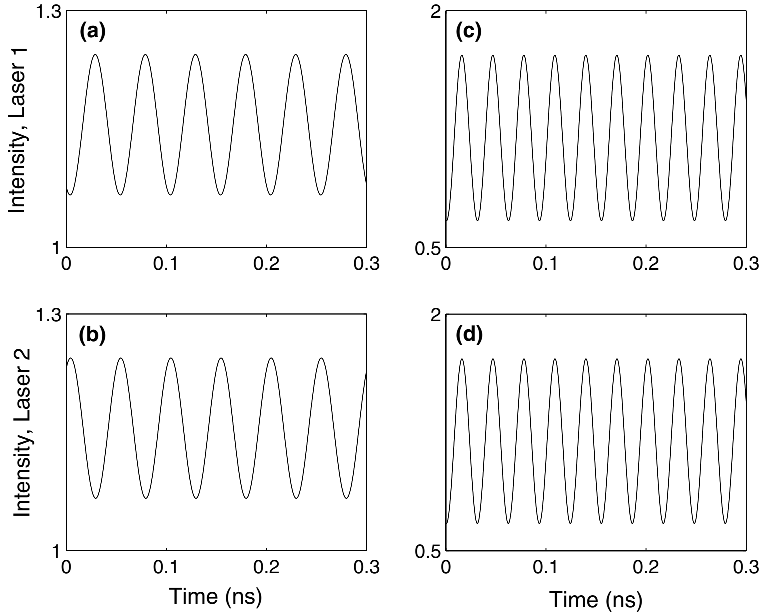

Fig. 5. (a,b) Time traces of the outputs of laser 1 and 2, respectively, for $\eta = 0.005$, $\delta = -20$ GHz and $\Omega\,\theta = 3$. (c,d) Same but for $\eta = 0.05$ and $\delta = -30.5$ GHz.

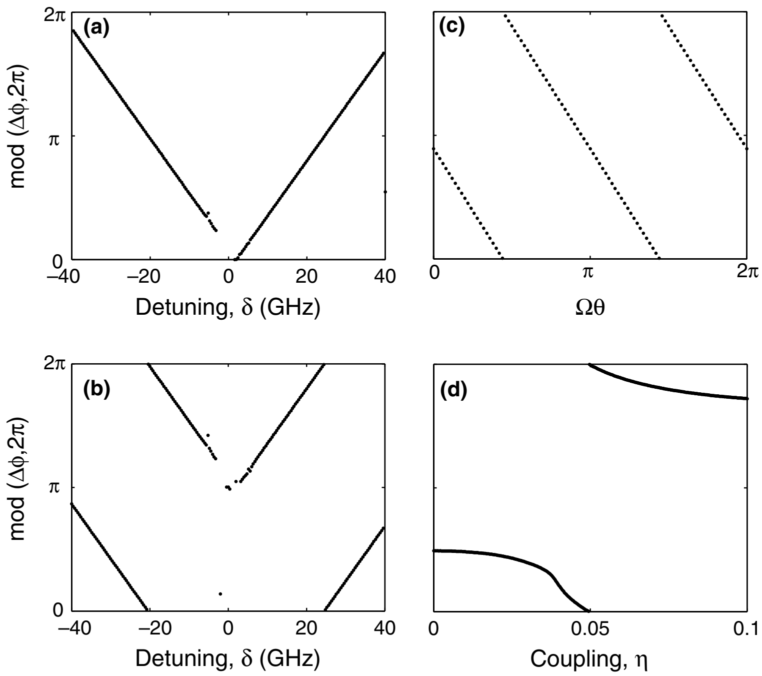

Fig. 6. Relative Hilbert phase between the periodically oscillating laser intensities as function of detuning for $\Omega\theta = 3$ (a) and $\Omega\theta = 3 + \pi/2$ (b) with $\eta = 5\times10^{-3}$. Relative Hilbert phase as function of the coupling phase $\Omega\theta$ for $\eta = 5\times10^{-3}$ and $\delta = 20$ GHz (c). Relative Hilbert phase as function of coupling η for $\Omega\theta = 3 + \pi/2$ and $\delta = 35.5$ GHz (d).

to the amplification of injected light and to four-wave mixing, respectively. Figs. 4(e) and (f) show representative four-wave mixing spectra. In the case of $\Omega\theta = 3$, the four-wave mixing regions are adjacent to the regions of injection-locked periodic solutions. However, the system undergoes a sudden transition as it crosses the boundaries between these regions.

Finally, we examine systematically the dependence of the relative phase between the periodically oscillating laser intensities on the coupling rate and the detuning frequency. For this purpose, we use the Hilbert transforms of the time series $I_1(t) = |E_1(t)|^2 - \langle|E_1|^2\rangle$ and $I_2(t) = |E_2(t)|^2 - \langle|E_2|^2\rangle$ where $\langle|E_i|^2\rangle$ is the mean intensity of laser i. These transforms read

$$H(I_i) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{I_i(\sigma)}{s - \sigma} \, \mathrm{d}\sigma, \tag{3}$$

with $i = 1$ or 2. The Hilbert phase $\phi_i(t)$ is the phase of the vector $(I_i(t), H(I_i(t)))$ in the complex plane. We define the relative Hilbert phase as $\Delta\phi(t) = \phi_2(t) - \phi_1(t)$ that is constant for periodic solutions. When the laser intensities oscillate perfectly in antiphase, $\Delta\phi = \pi$. By contrast, $\Delta\phi \rightarrow 0 \bmod 2\pi$ when they tend to oscillate in phase.

Figs. 2(c) and 3(c) show the dependence of the relative Hilbert phase on the coupling rate and the frequency detuning. As already noticed in [6] in absence of detuning, periodic solutions with identical amplitudes in lasers 1 and 2 of the form $E_1(s) = E_2(s - x)$, $N_1(s) = N_2(s - x)$ are necessarily either in phase with $x = 0$ or in antiphase with $x\Theta/2$, Θ being the normalized period. This can be readily shown by introducing these solutions in Eqs. (1) and (2). Periodic in phase solutions (i.e., with $x = 0$) are however not observed whenever the lasers start from non-identical initial conditions. The mappings of the relative Hilbert phase confirm that injection-locked periodic solutions oscillate perfectly in antiphase in the absence of detuning and that the detuning induces small deviation from perfect antiphase. The effect of the detuning decreases also with the coupling rate in these regions of the parameter space. By contrast, in the four-wave mixing regime, $\Delta\phi$ is strongly affected by the detuning, the coupling rate and the coupling phase. As examples, Fig. 5(a) and (b) show the intensities of the lasers oscillating perfectly in antiphase and Figs. 5(c) and (d) the intensities of the lasers oscillating perfectly in phase, respectively. The comparison between Figs. 2(b) and (c), on the one hand, and Figs. 3(b) and (c), on the other hand, suggests that there is a direct relationship between the Hilbert phase and the oscillation frequency and therefore the detuning for low coupling rates. Figs. 6(a) and (b) illustrate the modification of the relative Hilbert phase with respect to the detuning, the coupling rate being fixed at $\eta = 5\times10^{-3}$ for a coupling phase $\Omega\theta = 3$ and $3 + \pi/2$, respectively. The linear increase of $\Delta\phi$ with $|\delta|$ is evident. We note that perfect in phase and antiphase solutions occur for a detuning difference equal to half the external-cavity frequency $1/2\tau$ ($\cong$22.5 GHz). Fig. 6(c) shows $\Delta\phi$ as a function of the coupling phase, the coupling rate and the detuning being fixed at $\eta = 5\times10^{-3}$ and $\delta = -20$ GHz. The figure shows clearly that the relative Hilbert phase decreases linearly with $\Omega\theta$ and that perfect in phase and antiphase oscillations are observed for a $\pi/2$ shift in the coupling phase. The coupling rate affects also $\Delta\phi$ but in a more complicated way as illustrated in Fig. 6(d).

4. Conclusions

We have investigated the dynamics of two mutually coupled laser diodes with respect to frequency detuning, coupling rate and phase. Bifurcation maps reveal that the system follows a quasiperiodic route to chaos in most of the parameter ranges although period-doubling can also be observed at low coupling rate and for small detuning. There are several regions of stationary and periodic behaviors where the lasers are mutually injection-locked. In the injection-locked periodic regions, the modulation of the laser outputs results from the undamping of the relaxation oscillation at low coupling rate or from a beating between two pairs of single frequency rotating wave solutions at higher coupling rate. Solutions of the same nature were predicted in laser diodes with delayed optical feedback in the short cavity regime [11,12]. In addition, bifurcation cascades similar to those observed here for small detuning with stationary regions interspersed with regions of more complex behaviors were also reported in laser diodes with longer cavities but low pump current [10] and with short cavities and high pump current [12]. There are other periodic regions where the lasers are unlocked and their outputs are modulated almost at the detuning frequency in a similar way to laser diodes subject to constant, monochromatic optical injection [7–9]. Furthermore, the coupling phase has a profound influence on the location, the shape and the size of the parameters regions within which the different dynamics are observed. We have finally investigated the dependence of the relative Hilbert phase between the periodically oscillating laser intensities on the detuning frequency, the coupling rate and phase. We have in particular observed that, at low coupling rate, the relative Hilbert phase increases linearly with the absolute value of the detuning and that in phase and antiphase solutions occur for a detuning difference equal to the inverse of twice the coupling delay between the lasers.

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