PHYSICAL REVIEW E, VOLUME 64, 045203(R)

Two types of synchronization in unidirectionally coupled chaotic external-cavity semiconductor lasers

A. Locquet, F. Rogister, M. Sciamanna, P. Mégret, and M. Blondel
Advanced Research in Optics, Service d'Electromagnétisme et de Télécommunications, Faculté Polytechnique de Mons, 31 Boulevard Dolez, B-7000 Mons, Belgium

(Received 23 April 2001; published 21 September 2001)

We study numerically two distant unidirectionally coupled single-mode semiconductor lasers subject to coherent optical feedback. We show that two fundamentally different types of chaotic synchronization can occur depending on the strengths of the coupling and of the feedback of the receiver laser.

DOI: 10.1103/PhysRevE.64.045203 PACS number(s): 05.45.Xt, 42.55.Px, 42.65.Sf

Very shortly after the discovery of synchronized chaos [1], the use of this phenomenon to realize private communications was suggested [2]. The idea is to conceal an information-bearing message into the noiselike output of a chaotic transmitter and to exploit the synchronization of a receiver with the chaotic output of the transmitter to recover the information signal. We examine here the case of two unidirectionally coupled distant external-cavity chaotic laser diodes. These devices are well suited for high-speed communication networks based on optical fibers and their synchronization has been demonstrated experimentally by different groups [3–6]. Moreover, the presence of an optical feedback means that high-dimensional chaotic dynamics can be produced [7]. This property can be useful since the use of high-dimensional chaos has been proposed as a way of increasing the security level of chaotic optical cryptography [8]. In this Rapid Communication, we consider two identical single-mode semiconductor lasers, each having its own feedback loop created by an external mirror. The round-trip time in the external cavity of each laser is τ. It is the same for both lasers. We show that two different types of synchronization can occur if the two lasers are unidirectionally coupled. Depending on the operating conditions, the output of the receiver, at time t, synchronizes with the signal that is injected into it at the same time t or with the signal that will be injected into it at time $t+\tau$. The latter type of synchronization can be described as a form of anticipating synchronization as defined by Voss [9]. On the contrary, no phenomenon of anticipation is found in the former type of synchronization and we will refer to it as conventional synchronization.

The transmitter and receiver lasers are two very similar semiconductor lasers subject to weak to moderate coherent optical feedback. The lasers are assumed to be single mode even in the presence of external optical feedback. The unidirectional coupling is realized by optically injecting a fraction of the light produced by the transmitter into the active region of the receiver laser. We model the two lasers by using the rate equations for the amplitude $E(t)$ of the electric field, the slowly varying phase $\phi(t)$ of the electric field, and the carrier number $N(t)$. The presence of an optical feedback is taken into account by following the Lang and Kobayashi approach [10]. The rate equations for the transmitter laser (subscript T) are

$$\frac{dE_T(t)}{dt}=\frac{1}{2}\left\{\frac{g[N_T(t)-N_0]}{1+sE_T^2(t)}-\frac{1}{\tau_p}\right\}E_T(t)+\gamma_T E_T(t-\tau)\times\cos[(\omega_0\tau)_T+\phi_T(t)-\phi_T(t-\tau)], \tag{1}$$

$$\frac{d\phi_T(t)}{dt}=\frac{\alpha}{2}\left\{\frac{g[N_T(t)-N_0]}{1+sE_T^2(t)}-\frac{1}{\tau_p}\right\}-\gamma_T\frac{E_T(t-\tau)}{E_T(t)}\sin[(\omega_0\tau)_T+\phi_T(t)-\phi_T(t-\tau)], \tag{2}$$

$$\frac{dN_T(t)}{dt}=\frac{I}{e}-\frac{N_T(t)}{\tau_S}-\frac{g[N_T(t)-N_0]}{1+sE_T^2(t)}E_T^2(t). \tag{3}$$

The internal parameters of the laser are the gain parameter $g=1.5\times10^4\ \mathrm{s}^{-1}$, the gain saturation coefficient $s=5\times10^{-7}$, the carrier number at transparency $N_0=1.5\times10^8$, the linewidth enhancement factor $\alpha=5$, the photon lifetime $\tau_p=2$ ps, and the carrier lifetime $\tau_S=2$ ns. The operating parameters are the injection current $I=44$ mA (e is the elementary charge), the feedback rate $\gamma_T=2\times10^{10}\ \mathrm{s}^{-1}$, the external-cavity round-trip time $\tau=0.2$ ns and the phase mismatch after one round trip $(\omega_0\tau)_T=-2.5$ rad (mod 2π). Except for the phase mismatch, the parameter values are taken from Ref. [11]. For these parameter values, the laser operates in the chaotic regime of fully developed coherence collapse.

The receiver laser is subject to external optical feedback and to optical injection. The injection is described by adding a suitable forcing term to the standard laser equations [12]. We assume that no distortion is experienced by the electric field during its propagation between the transmitter and receiver lasers. Therefore, the electric field that is injected at time t into the receiver laser is proportional to the electric field that was produced by the transmitter laser at time $t-\tau_c$, where τ_c is the light propagation time between the output facet of the transmitter laser and the facet through which light is injected into the receiver laser. In the simulations, τ_c is chosen equal to 10 ns without loss of generality. We also assume that the parameters of the two lasers are identical except for the feedback rates (γ_T, γ_R) and for the solitary laser frequencies ($\omega_{0,T},\omega_{0,R}$). The dynamical behavior of the receiver laser (subscript R) is described by the rate equations

$$\frac{dE_R(t)}{dt}=\frac{1}{2}\left\{\frac{g[N_R(t)-N_0]}{1+sE_R^2(t)}-\frac{1}{\tau_p}\right\}E_R(t)+\gamma_R E_R(t-\tau)\times\cos[(\omega_0\tau)_R+\phi_R(t)-\phi_R(t-\tau)]+\gamma_{\rm ext}E_T(t-\tau_c)\cos[(\omega_{0,R}-\omega_{0,T})t+\phi_R(t)-\phi_T(t-\tau_c)+\omega_{0,T}\tau_c], \tag{4}$$

$$\frac{d\phi_R(t)}{dt}=\frac{\alpha}{2}\left\{\frac{g[N_R(t)-N_0]}{1+sE_R^2(t)}-\frac{1}{\tau_p}\right\}-\gamma_R\frac{E_R(t-\tau)}{E_R(t)}\sin[(\omega_0\tau)_R+\phi_R(t)-\phi_R(t-\tau)]-\gamma_{\rm ext}\frac{E_T(t-\tau_c)}{E_R(t)}\sin[(\omega_{0,R}-\omega_{0,T})t+\phi_R(t)-\phi_T(t-\tau_c)+\omega_{0,T}\tau_c], \tag{5}$$

$$\frac{dN_R(t)}{dt}=\frac{I}{e}-\frac{N_R(t)}{\tau_S}-\frac{g[N_R(t)-N_0]}{1+sE_R^2(t)}E_R^2(t), \tag{6}$$

where $\gamma_{\rm ext}$ is the external injection rate.

The aim of this Rapid Communication is to show that two fundamentally different types of synchronization can occur in unidirectionally coupled external-cavity semiconductor laser diodes depending on the values of the external injection rate $\gamma_{\rm ext}$ and of the receiver feedback rate γ_R. We assume in the following that there is no detuning between the two lasers ($\omega_{0,T}=\omega_{0,R}=\omega_0$). The first type of synchronization found [11,13], which we call conventional synchronization, corresponds to the fact that $\{E_R(t),\omega_0 t+\phi_R(t),N_R(t)\}$ tends to reproduce $\{aE_T(t-\tau_c),\omega_0(t-\tau_c)+\phi_T(t-\tau_c)+\Delta_\phi,N_T(t-\tau_c)+\Delta_N\}$, where a, Δ_ϕ, and Δ_N are constants. This means that the electric field produced by the receiver laser at time t synchronizes, up to a constant for its amplitude and phase, with the electric field that was produced by the transmitter laser at time $t-\tau_c$, which is also the electric field that is optically injected into the receiver at time t. This type of synchronization can be achieved if we take identical values for the feedback rates γ_T,γ_R and an external injection rate $\gamma_{\rm ext}$ that is sufficiently high. We take here $\gamma_T=\gamma_R=20\,{\rm ns}^{-1}$ and $\gamma_{\rm ext}=40\,{\rm ns}^{-1}$. It is possible to determine analytically that, for the parameter values chosen here, a is approximately equal to 1.016 [13]. Figure 1 shows the amplitude of the electric field injected into the receiver laser at time t multiplied by the constant factor a, $aE_T(t-\tau_c)$, and the amplitude of the electric field produced by this laser at the same time t, $E_R(t)$. We clearly see the synchronization between these two signals. The corresponding synchronization error, $aE_T(t-\tau_c)-E_R(t)$, is also represented in Fig. 1. We notice that the synchronization error is not equal to zero, which means that this form of synchronization is not perfect when the lasers are identical. However, if different photon lifetimes are assumed for the two lasers, a specific value of

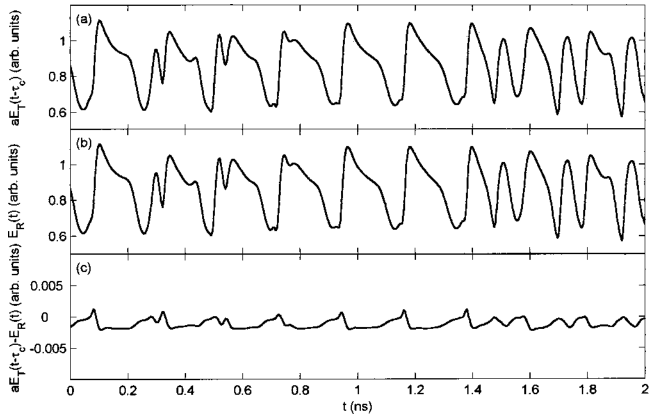

FIG. 1. Conventional synchronization. (a) Amplitude of the electric field injected into the receiver laser. (b) Amplitude of the electric field produced by the receiver laser. (c) Synchronization error.

the external injection rate, called the optimal injection rate [13], ensures a perfect synchronous solution.

Since $E_R(t)$ reproduces $aE_T(t-\tau_c)$ except for a small synchronization error, the derivatives of these two variables must be almost identical. Therefore, the sum of the terms of the right-hand side of rate equation (4) must be almost identical to the sum of the terms of the right-hand side of the shifted rate equation

$$\frac{d[aE_T(t-\tau_c)]}{dt}=\frac{1}{2}\left\{\frac{g[N_T(t-\tau_c)-N_0]}{1+sE_T^2(t-\tau_c)}-\frac{1}{\tau_p}\right\}aE_T(t-\tau_c)+\gamma_T aE_T(t-\tau-\tau_c)\cos[(\omega_0\tau)_T+\phi_T(t-\tau_c)-\phi_T(t-\tau-\tau_c)]. \tag{7}$$

We examine here how these different terms lead to equal sums and therefore to synchronization. The right-hand sides of the rate equations (4) and (7) contain a first term that corresponds to a competition between photon gain and loss mechanisms, a second term that takes into account the external optical feedback, and in the case of the receiver rate equation, a last term for the optical injection. These terms are represented in Fig. 2 after synchronization has occurred. We clearly see that the feedback term of the receiver is equal to the feedback term of the transmitter. On the contrary, the terms expressing a competition between photon gain and loss mechanisms are different in (4) and (7). This is principally due to the existence of a constant difference Δ_N between $N_T(t-\tau_c)$ and $N_R(t)$. It can easily be shown that this constant difference Δ_N implies that the difference between the two terms expressing a competition is almost proportional to $E_T(t-\tau_c)$. The injection term in (4) is also approximately proportional to $E_T(t-\tau_c)$ because of the synchronization, up to a constant $\Delta\phi$, of $\phi_R(t)$ with $\phi_T(t-\tau_c)$. In a synchronized state, the role of the injection term is to compensate for the difference between the terms expressing a competition between gain and loss mechanisms as can be seen in Fig. 2. The same type of compensation occurs in the phase rate equations. Simulations have been performed for different pa-

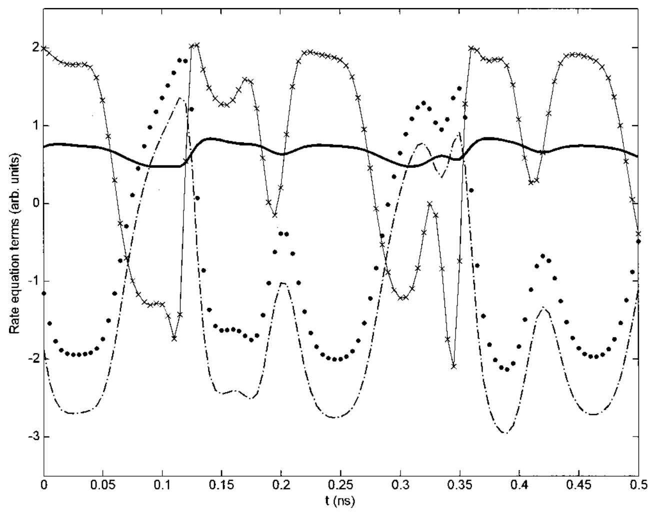

FIG. 2. Conventional synchronization. Contributions to the evolution of $aE_T(t-\tau_c)$ given by Eq. (7) and $E_R(t)$ given by Eq. (4): first term of the right-hand side of Eq. (7) (dots); feedback term in Eq. (7) (crosses); first term of the right-hand side of Eq. (4) (dash-dot); feedback term in Eq. (4) (thin solid line); external injection term in (4) (thick solid line).

rameter values than those given in this rapid communication and the same phenomena were observed. A simpler synchronization scheme in which the receiver laser is subject only to optical injection has been proposed [4,6]. With this scheme, if conventional synchronization can occur in the single-mode case, it cannot be described by the above mechanism since the receiver has no feedback term ($\gamma_R=0$).

The other type of synchronization corresponds to the synchronization of $\{E_R(t),\omega_0 t+\phi_R(t),N_R(t)\}$ with $\{E_T[t-(\tau_c-\tau)],\omega_0 t-\omega_0\tau_c+\omega_0\tau+\phi_T[t-(\tau_c-\tau)],\ N_T[t-(\tau_c-\tau)]\}$ as presented in Ref. [7]. It can be easily shown from Eqs. (1)–(6) that, in the absence of spontaneous emission noise, this synchronized solution exists if

$$\gamma_T=\gamma_R+\gamma_{\mathrm{ext}}. \tag{8}$$

We insist on the fact that this condition ensures the existence of the synchronized solution but does not tell anything about its stability. Physically, the electric field produced by the receiver laser at time t synchronizes with the electric field produced by the transmitter at time $t-(\tau_c-\tau)$, which is the field that will be optically injected into the receiver laser at time $t+\tau$. Voss has shown [9] that for certain unidirectional coupling configurations of two chaotic systems, the driven system anticipates the driver by synchronizing with one of its future states. The synchronization presented here corresponds to the same type of phenomenon when a propagation time τ_c between the two systems is taken into account. Therefore, we call the second type of synchronization ''anticipating synchronization'' as we have suggested in Ref. [14]. Masoller has pointed out in Ref. [15] that the receiver anticipates the electric field produced by the transmitter only if $\tau_c<\tau$ but we emphasize the fact that the synchronization type is the same whatever the value of τ_c: for any value of τ_c, the electric field produced by the receiver laser synchronizes with the field that will be injected into it τ seconds later and not, as in conventional synchronization, with the field that is injected at the same time. So the second type of synchronization is always anticipating with respect to the injected field. Figure 3 displays a simulation of the synchronization scheme for $\gamma_T=20\,\mathrm{ns}^{-1}$, $\gamma_R=5\,\mathrm{ns}^{-1}$, and $\gamma_{ext}=15\,\mathrm{ns}^{-1}$. It appears clearly that the output of the receiver $E_R(t)$ anticipates the signal that is injected into it, $E_T(t-\tau_c)$, with an anticipation time equal to $\tau=0.2\,\mathrm{ns}$. Figure 3 also presents the synchronization error, defined here as $E_T[t-(\tau_c-\tau)]-E_R(t)$; contrary to Fig. 1, the synchronization is perfect. However, we have noticed that for feedback rates larger than 10 ns^{-1} approximately, the synchronization quality degrades dramatically because of the instability of the synchronous solution. This behavior of perfect anticipating synchronization has also been found numerically in uni-

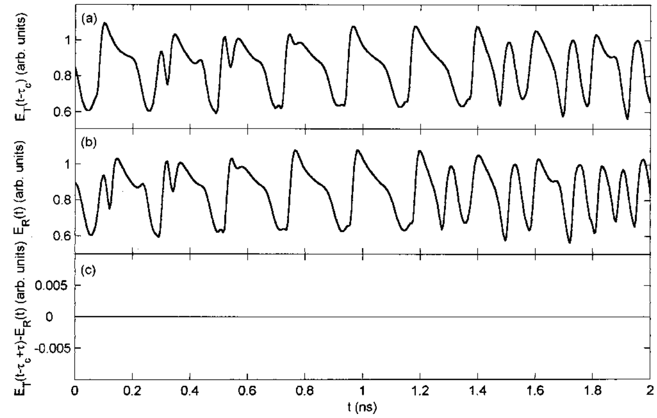

FIG. 3. Same as Fig. 1 for the case of anticipating synchronization.

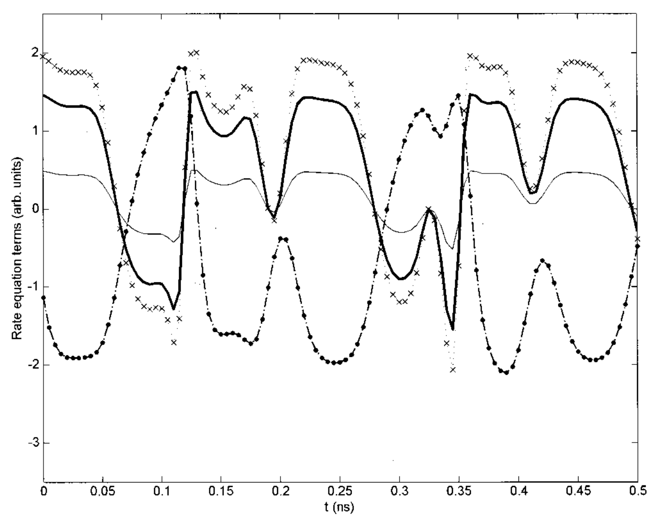

FIG. 4. Anticipating synchronization. Contributions to the evolution of $E_T(t-\tau_c+\tau)$ given by Eq. (9) and $E_R(t)$ given by Eq. (4): first term of the right-hand side of Eq. (9) (dots); feedback term in Eq. (9) ($\cdots\times\cdots$); first term of the right-hand side of Eq. (4) (dash-dot); feedback term in Eq. (4) (thin solid line); external injection term in (4) (thick solid line). The origin of time is not the same as in Fig. 2.

directionally coupled semiconductor lasers subject to incoherent optical feedback as shown in Ref. [16].

In this type of synchronization, since $E_R(t)$ synchronizes with $E_T[t-(\tau_c-\tau)]$, the sum of the terms of the right-hand side of the rate equation (4) must be equal to the sum of the terms of the right-hand side of

$$\frac{dE_T(t-\tau_c+\tau)}{dt} = \frac{1}{2}\left\{\frac{g[N_R(t-\tau_c+\tau)-N_0]}{1+sE_T^2(t-\tau_c+\tau)} - \frac{1}{\tau_p}\right\} \times E_T(t-\tau_c+\tau) + \gamma_T E_T(t-\tau_c)\cos[(\omega_0\tau)_T + \phi_T(t-\tau_c+\tau) - \phi_T(t-\tau_c)]. \quad (9)$$

The different terms of (4) and (9) are represented in Fig. 4. This time, the terms expressing a competition between gain and loss mechanisms are identical. The external injection term is proportional to the two feedback terms because of the synchronization of $E_R(t-\tau)$ with $E_T(t-\tau_c)$ and of $\phi_R(t-\tau)$ with $\phi_T(t-\tau_c)-\omega_0\tau_c+\omega_0\tau$. When condition (8) is met, the feedback term of the transmitter is equal to the sum of the receiver feedback and injection terms as can be seen in Fig. 4. The same mechanism occurs in the rate equations of the slowly varying phase. Anticipating synchronization in the case where the receiver is subject only to external optical injection can be explained exactly in the same way. It corresponds to the special case for which condition (8) is met by choosing $\gamma_{\rm ext}=\gamma_T$ and $\gamma_R=0$: the receiver feedback term vanishes and the injection term alone compensates for the transmitter feedback term. This configuration has been studied numerically in [17].

In practice, the quality of the synchronization is degraded by spontaneous emission noise, by a mismatch between the parameters of the two lasers, or in the case of chaotic secure communication, by an information-bearing signal that is mixed with the chaotic output of the transmitter laser. A quantitative study of the influence of a mismatch between internal parameters has been reported in Ref. [11] for conventional synchronization. In the case of anticipating synchronization, our simulations show that when spontaneous emission noise or small relative differences between parameters (1%) are taken into account, brief bursts of desynchronization appear in the synchronization error but, except for these bursts, the phenomenon of anticipating synchronization is clearly preserved.

In conclusion, we have shown that the chaotic output of an external-cavity semiconductor laser can be reproduced by an external-cavity receiver laser via two fundamentally different types of synchronization, conventional and anticipating synchronization. The type of synchronization obtained depends on the receiver feedback and external injection rates.

A.L. thanks Professor P. Mandel for valuable comments. This research has been supported by the Inter-University Attraction Pole program (IAP IV/07) of the Belgian government. M.S. received financial support from the Fonds National de la Recherche Scientifique.

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