Properties of the pulse train generated by repetition-rate-doubling rational-harmonic actively mode-locked Er-doped fiber lasers

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We demonstrate for the first time to our knowledge, experimentally and theoretically, that the pulse-to-pulse amplitude fluctuations that occur in pulse trains generated by actively mode-locked Er-doped fiber lasers in a repetition-rate-doubling rational-harmonic mode-locking regime are completely eliminated when the modulation frequency is properly tuned. Irregularity of the pulse position in the train was found to be the only drawback of this regime. One could reduce the irregularity to a value acceptable for applications by increasing the bandwidth of the optical filter installed in the laser cavity.

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Rational harmonic mode locking (RHML) is attractive for its potential to multiply the repetition rate of a pulse train generated by actively mode-locked Er-doped fiber lasers.[1] Although the pulse train's repetition rate is increased only twofold with respect to modulation frequency f_M in a repetition-rate-doubling (RRD) RHML regime, the best pulse-train quality was obtained in this regime. Indeed, in the RRD RHML regime, ~35-dB suppression of an unmatched component at modulation frequency f_M in the rf spectrum of the pulse train was observed experimentally[2] when the modulation frequency detuning, $\delta f_M = f_M/f_{\mathrm{FSR}} - n - 1/2$, was equal to zero ($f_{\mathrm{FSR}}$ is the free spectral range of the laser cavity; n is an integer). Although a theory of RHML has been developed,[3] the nature of the f_M component is not completely elucidated, and ultimate limitations on the parameters of the pulse train in the RRD RHML regime are not determined.

In this Letter a detailed experimental and theoretical investigation of the parameters of the pulse train in the RRD RHML regime of Er-doped fiber lasers is presented for the first time to the authors' knowledge. For clarity we consider a dispersion-compensated laser cavity and do not take into account Kerr nonlinearity in the optical fiber. In the experiment, the relative temporal position of the pulse circulating in the laser cavity (with respect to cavity-loss modulation), pulse-to-pulse amplitude fluctuations, and pulse width were measured. The theoretical model is based on the self-consistency of the pulse after two round trips in the laser cavity, and an analytical solution is obtained for a dispersion-compensated Er-doped fiber laser in the RRD RHML regime.

For theoretical analysis we consider an actively mode-locked fiber laser in a unidirectional ring configuration. The laser cavity consists of a Mach–Zehnder modulator (MZM) followed by an output coupler, a Fabry–Perot filter (FPF), and an Er-doped fiber as the amplification medium. All elements are connected by optical fiber to form a ring. It can be shown that only the total cavity dispersion, rather than the dispersion of each individual piece of optical fiber, influences the parameters of the pulse train. As we mentioned above, we consider the case of a dispersion-compensated cavity. The saturated gain of the Er-doped fiber is assumed to be equal to intracavity loss, spectrally flat within the FPF's transmission bandwidth, and time independent. The transfer function of the FPF is $F(v) = 2F_0F_1\{F_0 + F_1 - (F_0 - F_1)\exp[-2\pi i(v - v_0)/v_{\mathrm{FSR}}]\}^{-1}$, where v is the optical frequency, v_0 is the optical frequency at maximal transmission of the FPF, v_{FSR} is the FPF's FSR, and F_0 and F_1 are the FPF's maximum and minimal transmissions, respectively. The finesse of the FPF is $k_F = v_{\mathrm{FSR}}/\Delta v$, where Δv is the FPF's FWHM bandwidth. The time-dependent transfer function of the MZM is $M(t) = M_0 \sin[\psi_0 + \pi R \sin(\theta)]$, where $\theta = 2\pi f_M t$ is the normalized time, M_0 is the MZM's maximal transmission, ψ_0 is the phase factor, $R = V_M/V_\pi$, V_π is the π voltage of the MZM, and V_M is the amplitude of the modulation voltage. For RRD RHML, two conditions must be satisfied: $0 \le \psi_0 \le \pi/2$ and $0 \le \pi R \le \min(\psi_0; \pi/2 - \psi_0)$. These conditions exclude other repetition-rate multiplication mechanisms, such as use of the nonlinear transfer function of the MZM through appropriate MZM biasing[3] or adjustment of the modulating signal amplitude.[4] In this Letter, for our theoretical analysis we use the anharmonic transmission function of MZM because we intend to compare experimental and theoretical dependencies. However, qualitatively the same results can be obtained for a purely harmonic modulation function. This means that high-order harmonic terms in the Fourier expansion of the MZM transfer function are not crucial for RRD RHML pulse formation when the conditions mentioned above are valid.

For an analysis of RRD RHML we apply the theory of Kuizenga and Siegman,[5] assuming a Gaussian pulse circulating inside the laser cavity. Both the MZM's transmission in time and the FPF's transmission in frequency are approximated by Gaussian functions by use of quadratic

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expansions. A self-consistent steady-state solution is obtained after two complete cavity round trips. For two sequential cavity round trips, the electrical field of the optical pulse at the MZM's output is expressed in the following form: $E_m(t) = E_{0m}\exp[2\pi i v_0(t - t_m)]\exp[-2\ln 2(t - t_m)^2/\tau_m{}^2]$, where $m = 1, 2$ correspond to the first and second cavity round trips, respectively, E_{0m} is the complex amplitude of the pulse, t_m is the temporal position of the pulse peak, and τ_m is the FWHM pulse width. Normalized time $\theta_0 = \pi[(t_2 + t_1)/T_M - n - 1/2]$ is introduced, which represents the temporal position of the middle point between pulse positions at the first and the second round trips. The temporal shift of the pulse from this middle point is described by dimensionless parameter $\Delta\theta = \pi[(t_2 - t_1)/T_M - n - 1/2]$. Pulse-to-pulse amplitude fluctuation is characterized by parameter $\delta I = (|E_{02}|^2 - |E_{01}|^2)/(|E_{02}|^2 + |E_{01}|^2)$. Simple analytical expressions can be obtained in first order on small parameter $k = f_M/\Delta v$ for pulse-train parameters at the MZM output, namely,

$$\Delta\theta = -\frac{\pi k}{k_S}\frac{R\sin(2\psi_0)\cos(\theta_0)}{2a\sqrt{\alpha_{m12}}}, \tag{1}$$

$$\delta I = -\cot(\psi_0)\tan[\pi R\sin(\theta_0)] \times \left[1 - \frac{k}{k_S}\frac{\pi^2 R^2\cos^2(\theta_0)}{a\sqrt{\alpha_{M12}}}\right], \tag{2}$$

$$\tau_m = \frac{1}{\pi}\left(\frac{2\ln 2}{k_S f_M \Delta v\sqrt{\alpha_{M12}}}\right)^{1/2}\left(1 - \frac{k}{k_S}\frac{\alpha_{Mm}}{\sqrt{\alpha_{M12}}}\right), \tag{3}$$

where $k_S = \sin(\pi k_F/2)/(\pi k_F/2)$, $a = M(\theta_0)M(\theta_0 + \pi)/M_0{}^2$, $\alpha_{M12} = \alpha_{M1} + \alpha_{M2}$, $\alpha_{M1} = \alpha_M(\theta_0)$, $\alpha_{M2} = \alpha_M(\theta_0 + \pi)$, $\alpha_M = -(1/2)[d^2\ln[M(\theta)]/d\theta^2\}$. Temporal parameter θ_0 can be calculated from the following transcendental equation:

$$\frac{k_S}{k}\delta f_M = \frac{R\cos(\theta_0)\sin[2\pi R\sin(\theta_0)]}{2a\sqrt{\alpha_{M12}}}. \tag{4}$$

For experimental investigation of the RRD RHML regime we used an actively mode-locked Er-doped fiber laser in a sigma configuration. A detailed description of the laser is given in Ref. 2. In the present experiment we compensated for intracavity dispersion by inserting a piece of dispersion-compensating fiber to be consistent with the theoretical model. All measurements were made at 1545 nm. The intracavity dispersion was estimated to be approximately 0.01 ± 0.01 ps/nm at 1545 nm by measurement of the dependence of the mode-locking frequency on the operational wavelength. The FSR of the laser cavity was ~1.567 443 MHz. The MZM used in experiment was optically biased in such a way that its optical transmission was approximately one half of the maximal transmission for zero voltage at its dc input and no signal at its rf input. Phase factor ψ_0 was measured to be $\sim 0.19\pi$. The tunable optical filter installed in the cavity was not a Fabry–Perot filter. Double-pass optical transmission of the actual filter was fitted by the function $|F(v)|^2$ to yield filter parameters are compatible with theory. Fitting the filter's transmission spectrum led to a FWHM bandwidth of $\Delta v = 2.2$ nm and $k_S = 1$. In the present experiment the value of the small parameter was $k = 0.065$. Laser output was detected by a photoreceiver with a 25-GHz bandwidth and monitored by a 20-GHz sampling oscilloscope. The oscilloscope was triggered by the modulating signal applied to the MZM. To facilitate measurement of the pulse timing in the cavity with respect to cavity-loss modulation, we introduced additional time delay to the triggering signal. This additional delay was tuned to equalize the total delay of the measured signal (optical delay plus electrical delay after the photoreceiver) and the delay of triggering signal. To determine the instant corresponding to maximal transmission of the MZM, we tuned the modulation frequency for harmonic mode locking to $f_M \approx 2.699\,614\,627$ GHz. For optimally tuned harmonic mode locking, pulses circulating in the cavity are synchronized with instants of the MZM's maximal transmission, providing reference points in the oscilloscope trace. In the theoretical model presented, these instants correspond to $\theta = \pi/2 + 2\pi l$, where l is an integer. Then we detuned the modulation frequency by half of the laser cavity's FSR to achieve the RRD RHML regime.

Oscilloscope traces were recorded for several values of the modulation frequency detuning δf_M. Pulse-to-pulse amplitude fluctuation δI and temporal parameters θ_0 and $\Delta\theta$ were measured from the oscilloscope traces. The resultant experimental dependencies are shown in Figs. 1 and 2 for normalized modulation amplitudes $R = 0.07, 0.10, 0.14$. The theoretical dependencies calculated for actual laser parameters are shown in the same figures. Experimental and theoretical dependencies of the pulse duration (averaged on two round trips) and the time–bandwidth product on the normalized modulation amplitude for $\delta f_M = 0$ are shown in Fig. 3. Quite

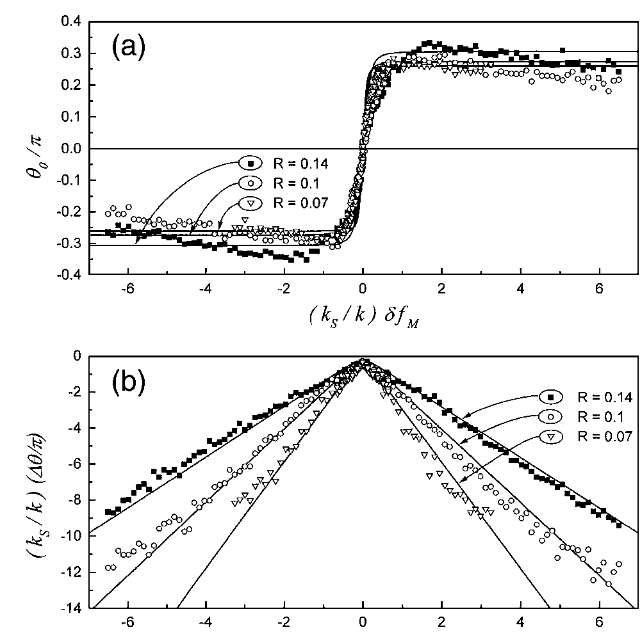

Fig. 1. Experimental and theoretical dependencies of temporal parameters (a) θ_0 and (b) $\Delta\theta$ on the normalized modulation frequency detuning.

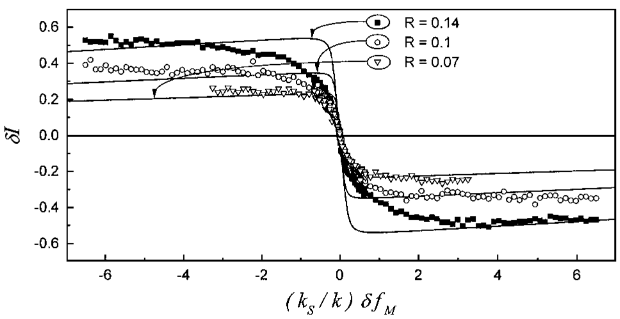

Fig. 2. Experimental and theoretical dependencies of pulse-to-pulse amplitude fluctuations δI on the normalized modulation frequency detuning.

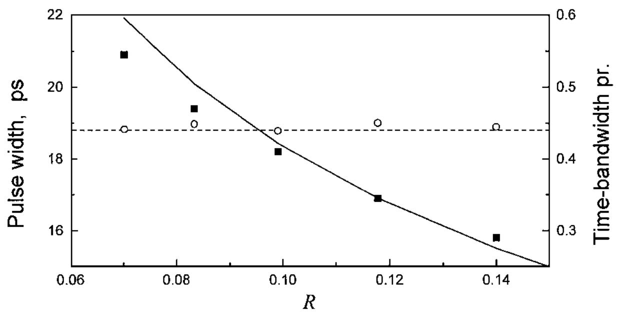

Fig. 3. Experimental and theoretical dependencies of the pulse width (filled squares and solid curve) and the time–bandwidth product (open circles and dashed line) on the normalized modulation amplitude for $\delta f_M = 0$.

good agreement between experimental results and the theoretical model is observed. Some discrepancy is due to the increasing error of the theoretical model with increasing modulation frequency detuning. Furthermore, the pulse train generated became unstable and noisy for large modulation frequency detuning, which increased the experimental error for large values of the detuning.

The main properties of the pulse train in the RRD RHML regime are summarized in what follows. According to our approach and as has been experimentally confirmed, the pulse train results from the superposition of two temporally shifted pulse trains, both at the same repetition rate f_M. The temporal shift between these pulse trains is approximately half of the modulation period $T_M = 1/f_M$. Therefore the repetition rate of the resultant pulse train is $2f_M$. A component at f_M appears in the rf spectrum if any of the pulse parameters is different at the first and the second round trip or if the temporal shift between the two superposed trains is different from the ideal value $T_M/2$.

The instants at which the circulating pulse passes through the modulator at the first and the second round trips depend on δf_M, as shown in Fig. 1(a). For any value of δf_M, however, the pulse never passes through the modulator when its transmission is maximal or minimal, as was assumed in Ref. 1. When $\delta f_M = 0$ the pulse is timed near $\theta = 0$ and $\theta = (2n + 1)\pi$ at the first and second round trips, respectively; i.e., at both the first and the second round trips, the pulse passes through the modulator when its transmission is near the middle between minimum and maximum. A simple physical explanation of this pulse timing can be found in the time domain. In the RRD RHML regime, an optical pulse circulating in the cavity is self-consistent after two round trips, so we could analyze the time-dependent transmission of the modulator that a pulse experiences after two sequential round trips. Pulse timing in the laser cavity at the first and the second cavity round trips that is obtained theoretically and observed in experiment corresponds to approximately the maximum of the dual round-trip MZM transmission condition favors the formation of RRD RHML pulses.

The temporal shift between the two superposed pulse trains, $T_{21} = t_2 - t_1 - nT_M$, is always different from its optimal value $T_M/2$. The higher the value of δf_M, the higher the difference $\Delta T = T_{21} - T_M/2 = T_M \Delta\theta/\pi$ [Fig. 1(b)]. When $\delta f_m = 0$, the temporal offset ΔT is minimal and $\Delta T = -\cos(\psi_0)/(\pi k_S \Delta v)$. The resultant pulse train suffers from irregularity of the relative temporal position of the pulses in the train. It appears as a periodic alternation of the $T_M/2 - \Delta T$ and $T_M/2 + \Delta T$ temporal distances between sequential pulses in the train. The irregularity is strongly reduced when $\delta f_M = 0$, although it cannot be completely eliminated and causes a residual component at modulation frequency in the rf spectrum of the pulse train. Nevertheless, one may reduce this irregularity by increasing the optical filter bandwidth, which in turns also reduces the pulse duration.

The pulse amplitudes at the first and the second round trips depend on δf_M, as shown in Fig. 2, and, in general, are not equal. However, they are strictly equal when $\delta f_M = 0$. This means that pulse-to-pulse amplitude fluctuations are completely suppressed when $\delta f_M = 0$. This result is important and contrasts with the general belief that pulse-to-pulse amplitude fluctuations are unavoidable in RHML.

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