Experimental study of supermode noise of harmonically mode-locked erbium-doped fibre lasers with composite cavity

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Abstract

An experimental study of supermode noise of harmonically actively mode-locked erbium-doped fibre lasers in the composite-cavity configuration is reported. Our study shows that the number of supermodes is reduced, as expected. However, this reduction is by far less efficient than what was previously reported. In particular, we found in measured RF spectra that supermodes appear in wide periodic frequency windows. Our most significant result is that, contrary to what has been previously reported, we did not measure in the composite-cavity laser any significant reduction of the overall supermode noise in comparison with the standard single-cavity configuration. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Harmonically and actively mode-locked erbium-doped fibre lasers are stable sources of picosecond pulses at GHz repetition rates that are useful in applications like high-speed telecommunications and photonic analog-to-digital conversion. Usually, due to the long length of ring laser cavity, the fundamental cavity resonance frequency (or free spectra range, FSR) lies in the MHz range. Hence, in order to obtain GHZ repetition rates, mode locking is achieved at a very high harmonic of the FSR. As a result, a large number of supermodes compete with each other and beating between them leads to large pulse amplitude fluctuations \cite{1} and phase noise \cite{2}. Several techniques have been proposed to reduce supermode noise. Harvey and Mollenauer \cite{1} used a high-finesse Fabry–Perot étalon filter, selecting only one supermode, but requiring a stabilization scheme. In \cite{3}, dithering of the cavity length at kilohertz rate is thought to reduce spatial hole

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burning, resulting in a reduction of the unwanted supermodes, but this effect was not clearly understood. Other techniques demonstrated the use of intensity-dependent loss to reduce pulse amplitude fluctuations. Additive pulse limiting (APL) [4] relies on non-linear polarization rotation in the presence of a polarizer. Because APL needs a non-polarization-maintaining cavity, its implementation makes the laser sensitive to environmental perturbations. The combined effect of self-phase modulation (SPM) and optical filtering [5] is a simple technique to implement, but several hundreds of metres of fibre are required to get a sufficient amount of SPM. Pulse intensity limiting in the form of two-photon absorption by a semiconductor mirror is another technique which was recently demonstrated [6]. This technique however does not rely on fibre-optics components. Finally, the composite cavity was proposed to reduce supermode beat noise in harmonically mode-locked erbium fibre lasers [7]. Among all these techniques, the latter appears as the most versatile. The composite cavity is formed by two sub-cavities whose absolute lengths can be arbitrarily long. The path length difference between the two cavities, say $\Delta L$, is adjusted so that their modes only coincide at large multiples of their FSR. In this way the number of effective modes of the cavity can be made very small and, hence, supermode noise is in principle drastically reduced [7]. In this paper, we report on a systematic experimental study of the composite-cavity laser with a wide range of $\Delta L$ values. RF spectrum measurements of the modelocked pulse train show that the composite-cavity configuration is not as efficient as what is stated in [7] for reducing the number of supermodes. Furthermore, signal-to-noise ratio calculated from these spectra clearly reveal that no significant reduction of the overall supermode noise is obtained.

2. Experiment

We use an environmentally stable and harmonically mode-locked erbium-doped fibre sigma laser (Fig. 1) [8]. The sigma cavity [9] is functionally equivalent to a unidirectional polarization-maintaining (PM) ring cavity. The PM property of the cavity is very important, as it enhances considerably the laser stability and ensures reproducibility of the measurements. Moreover, the cavity length of our laser is stabilized against environmental perturbations (temperature changes, mechanical vibrations) thanks to the use of a

![Fig. 1. Harmonically mode-locked erbium fibre laser with composite cavity; EDFA: erbium-doped fibre amplifier; TOF: tunable optical filter; PBS: polarization beam splitter; FRM: Faraday rotation mirror; PZT: piezoelectric transducer. The basic laser has a sigma cavity. The composite cavity is implemented by inserting a Mach–Zehnder fibre interferometer in the ring (dashed box). For environmental stability of the laser, the cavity length is controlled through a piezoelectric transducer on which single-mode fibre is wound. Principle and implementation of the stabilization feedback loop are explained in [8].]
feedback loop based on the minimization of the average inter-pulse noise that is measured at the second output of a dual-output Mach–Zehnder electro-optic modulator. The net cavity dispersion is measured to be 8.6 ps/(nm km). The center wavelength is set to 1550 nm by the use of a 3 nm tunable optical filter in the non-PM branch of the laser. The composite-cavity configuration is implemented by inserting in the PM ring of the sigma laser a Mach–Zehnder fibre interferometer that is made of two 50/50 PM couplers and PM fibre pigtails of different lengths (dashed line in Fig. 1). Length of the shorter sub-cavity is equal to \( L_1 \approx 125 \text{ m} \) in all cases. We measure the supermode noise of the composite-cavity laser in a wide range of values of \( \Delta L = L_2 - L_1 \), from a few centimetres up to several tens of metres. The following values of \( \Delta L \) are considered: 6.8 cm, 20.0 cm, 33.5 cm, 1.2 m, 3.4 m, 13.0 m and 28.4 m. For comparison, we also measure the supermode noise of the single-cavity laser \( (L_1 \approx 125 \text{ m}) \), obtained by disconnecting the longer arm of the interferometer. The laser is harmonically mode-locked at \( f_m \approx 3 \text{ GHz} \). Through background-free optical autocorrelation, the pulse width is measured to lie in the range 10–15 ps for all configurations. The average optical output power of the laser is about 1.6–2.0 mW in all cases. We believe that no soliton pulse shaping is occurring in the cavity. The pulse train is detected using a 20 GHz photodiode connected to either an electrical spectrum analyser (22 GHz bandwidth) or a sampling scope (20 GHz bandwidth). In order to quantify supermode noise, we analyze both RF spectra and scope traces.

3. Results

Fig. 2 shows the RF spectra of mode-locked pulse trains measured between \( f_m \) and \( 2 \times f_m \) for the single-cavity laser and for the composite-cavity laser with several values of \( \Delta L \). Given such a wide frequency span, the resolution was not sufficient to resolve individual supermodes, and only their envelope is visible. In the single-cavity case [Fig. 2(a)], as expected, all supermodes are present between \( f_m \) and \( 2 \times f_m \) with a roughly constant power level in the whole frequency range. In the composite-cavity cases [Fig. 2(b)–(d)], groups of supermodes are observed within periodic frequency windows whose period is equal to \( \Delta \nu_L = c/(n\Delta L) \), where \( c \) is light velocity in vacuum and \( n \) is fibre effective index. As it could have been expected, \( \Delta \nu_L \) corresponds to the periodicity of the transmittance of the intracavity Mach–Zehnder fibre interferometer. Indeed, such an unbalanced

![Fig. 2. RF spectra of mode-locked pulse trains measured between the first and second harmonics of modulation frequency \((f_m \approx 3 \text{ GHz})\). Resolution = 300 kHz. Signal-to-noise ratio, SNR (in dB in figure) is defined in text: (a) single-cavity laser; (b) composite-cavity laser with \( \Delta L = 6.8 \text{ cm} \); (c) composite-cavity laser with \( \Delta L = 20.0 \text{ cm} \); (d) composite-cavity laser with \( \Delta L = 33.5 \text{ cm} \).](image-url)
Mach–Zehnder interferometer can be regarded as an optical filtering component, whose FSR is equal to $\Delta v_i$ [10]. In this sense, the composite-cavity approach is quite similar to the insertion of a high-finesse Fabry–Perot étalon in the cavity to select one supermode [1], with the difference however that the finesse $F$ of the Mach–Zehnder interferometer filter, defined as the ratio of its FSR to its bandwidth at half maximum, is low ($F = 2$, independently of the FSR, $\Delta v_i$). Between these windows defined by the interferometer filter, the supermode power level is below the noise floor of the detection set-up. We observe in Fig. 2 that the width of the supermode noise windows, like their separation $\Delta v_i$, is inversely proportional to their number $N$ between $f_m$ and $2 \times f_m$, i.e., $N = f_m/\Delta v_i = f_m \times n \times \Delta L/c$. This is consistent with the constant finesse of the Mach–Zehnder interferometer filter. Note that the actual width of these windows is smaller than the bandwidth of the single-pass Mach–Zehnder interferometer filter, because the photon lifetime in the laser is longer than the cavity round-trip time. Because they are only present in periodic frequency windows, the number of supermodes of the composite cavity is clearly reduced in comparison with the single cavity. However, we note that, quite remarkably, the maximal power level of supermodes of the composite cavity is systematically higher than the constant power level of supermodes of the single cavity (Fig. 2). So, it appears that supermode noise power tends to concentrate at frequencies where supermodes are allowed to oscillate. In the following, we estimate the supermode contribution to the signal-to-noise ratio (SNR) of the composite cavity by calculating the ratio between the carrier power level at $f_m$ and the average power level of supermodes included between $f_m$ and $2 \times f_m$ (see Fig. 2). With this procedure we extend the usual definition of SNR to the case of non-uniform supermode power spectral distributions. The SNR calculated from RF spectra (taken on shorter spans and with higher resolution than in Fig. 2) is rather surprisingly found to be similar in all cases: $\text{SNR} = 42.9, 41.5, 41.7, 41.3, 40.7, 37.5$ and $40.4$ dB for the composite cavity with the above mentioned values of $\Delta L$ (in growing order), while we measure $\text{SNR} = 42.7$ dB for the single cavity.

Note that the latter value is better by about 15 dB than the one obtained in [7] in the single-cavity configuration. We attribute this difference to the fact that we use both a PM cavity and a cavity-length stabilization scheme, contrary to [7]. We believe however that non-linear effects in the cavity were insufficient for the supermode noise reduction mechanism described in [5] to take place.

The differences between the SNR values given in the above are not significant. This result is confirmed by our observation of the oscilloscope traces. Indeed, pulse intensity fluctuations can be detected on an oscilloscope as a scattering of the pulse peak in the infinite persistence mode. The traces we observed showed a large scattering in both single- and composite-cavity configurations, and were very similar to those observed in [11] in the linear regime of an actively mode-locked erbium fibre laser subject to supermode beat noise. Using the statistical tools of the sampling scope, we recorded the density distribution of the pulse peak in infinite persistence mode. These are presented in Fig. 3 for the single cavity and the composite cavity in the case $\Delta L = 6.8$ cm. It appears from the figure that these distributions are very similar. The peak intensity distributions of all other composite-cavity configurations were found to be similar to those of Fig. 3. By measuring the mean and the standard deviation of these distributions, we determined that the root mean square (rms) amplitude noise was about 40% in all cases, which shows again that the composite-cavity configuration did not yield any significant reduction in the supermode noise. Note however that this large noise value does not include only supermode amplitude fluctuations, but also other noise contributions that occur at different frequencies, like relaxation oscillations, or low-frequency amplitude noise, as the analysis of a scope trace does not allow any discrimination between all these components of amplitude noise. Note finally that, contrary to the rather large 40 dB SNR calculated from RF spectra, which only takes into account the power level of one single unwanted supermode, the 40% amplitude variation measured with the scope contains contribution from all supermodes, whose number is large ($\sim 2000$).
Fig. 4 shows the RF spectrum of mode-locked pulsetrains measured over a short span around $f_m$ in the case $\Delta L = 13.0$ m. As $\Delta L$ is much larger (i.e., $\Delta V_f$ much smaller) than in the cases of Fig. 2, the width of the periodic frequency windows (which are caused by the transmittance of the Mach–Ze–nhnder interferometer filter, in dashed line in the figure) is small and only contains two, or at most three modes. Again, we find that the periodicity of the supermode noise windows is equal to $\Delta V_f = c/(n\Delta L)$, i.e., $\Delta V_f = 15.4$ MHz in this case. According to [7], the fundamental resonance frequency of the composite cavity should be equal to the lowest common multiple of the fundamental resonance frequencies of both sub-cavities, say $\Delta V_c$: $\Delta V_c = N_1 \times \Delta V_1 = N_2 \times \Delta V_2$, where $\Delta V_1$ and $\Delta V_2$ are the FSRs of the two sub-cavities. In the present case, $\Delta V_1$ and $\Delta V_2$ were measured to be 1.60 and 1.45 MHz, respectively, which yields $\Delta V_c = 76.8$ MHz ($N_1 = 48$ and $N_2 = 53$). $\Delta V_c$ is represented in Fig. 4. It shows evidence that $\Delta V_c$ is larger than the spacing between consecutive composite-cavity modes and is not, therefore, the fundamental resonance frequency of the composite cavity. This result shows again that the number of supermodes in the composite cavity is larger than what was previously suggested. Indeed, according to [7], and given the span of Fig. 4, only one mode should appear on the right side of the carrier (the mode labeled 1, corresponding exactly to a transmittance maximum). Although it is beyond the scope of this paper, one can show that $\Delta V_c$ is an integer multiple of $\Delta V_f$, the integer being higher than one in general (5 in this case). One can also show that, within the windows, supermodes are separated by $\Delta V_m = 2\Delta V_1\Delta V_2/(\Delta V_1 + \Delta V_2)$. In the present case, this formula yields $\Delta V_m = 1.52$ MHz, which is in perfect agreement with composite-cavity mode frequencies observed in Fig. 4.

The number of modes in each window in the RF spectrum of a pulse train generated by the composite cavity depends on the ratio $\Delta V_f/\Delta V_m$. In the case $\Delta L = 13.0$ m, we find that $\Delta V_f/\Delta V_m = 10.1$, so that this composite-cavity configuration is close to the configuration investigated in [7] ($\Delta V_f/\Delta V_m = 10.5$). As a consequence, one could expect that the same number of modes appear in each frequency window in Fig. 4, and in Fig. 3(b) of [7]. However, Fig. 4 shows evidence of two or three modes per window, whereas only one mode is observed in each window in [7]. We explain this difference as follows. In the particular configuration of [7], $\Delta L$ has been chosen precisely in order to verify $\Delta V_c = \Delta V_f$, so that one mode is found at the center of each window. In contrast, in the case of Fig. 4, $\Delta V_c = 5\Delta V_f$, so that modes are found at the center only once every five windows (this is the case of the mode labeled 1 in Fig. 4). As only these particular modes match the maximal transmittance of the Mach–Ze–nhnder interferometer, logically, they are larger than the modes on their
left- and right-hand sides (see the modes labeled 2 and 3 in Fig. 4). In all other windows, the modes do not perfectly match maximal transmittance. Two modes may be very close to that position, however, and appear together in the same window, with almost equal magnitudes if they are approximately equidistant on each side of the window center (see for example the modes labeled 4 and 5 in Fig. 4). Note that, in Fig. 4, the two lateral modes labeled 2 and 3 are about 30 dB below the central mode (labeled 1), slightly above the noise floor. In the RF spectrum presented in Fig. 3(b) of [7], the measurement noise floor is only 20 dB below the central mode of each window, so that lateral modes can not be observed, would they exist. Thanks to the presence of an adjustable delay line in the shorter branch of the Mach–Zehnder interferometer, we were able to tune precisely the value of $\Delta L$ in order to verify $\Delta v_m = \Delta v_f$, like in [7]. This condition was satisfied for $\Delta L = 12.5$ m. As expected, modes were observed exactly at the center of each window, but no improvement of the supermode noise suppression was measured.

4. Conclusion

We have studied supermode noise of harmonically mode-locked erbium-doped fibre lasers in the composite-cavity configuration. In measured RF spectra, supermodes are only detectable within periodic frequency windows, whose periodicity is determined by the path length difference of the composite cavity. The main outcome of our study is that, whatever the path length difference, we observed no significant reduction of the overall supermode noise by implementing the composite cavity, whereas the author of [7] did observe such reduction, using a different laser geometry. The difference between our results and those of [7] may be attributed to the fact that we are starting with a single-cavity laser having better supermode suppression. The improved suppression in our single-cavity laser is attributed to the use of an all-polarization-maintaining cavity in conjunction with cavity length stabilization. Further improvement in the supermode noise suppression by use of the composite-cavity technique may be limited by the low finesse of the unbalanced Mach–Zehnder interferometer.

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