Supermode Noise of Harmonically Mode-Locked Erbium Fiber Lasers With Composite Cavity

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***Abstract*—The supermode noise of a harmonically mode-locked erbium-doped fiber laser with composite cavity is investigated both theoretically and experimentally. We propose a simple model based on the transfer function of the composite cavity. From this model, the frequencies of the cavity modes and their frequency-dependent losses are determined. A comprehensive experimental study of the composite cavity laser is carried out, covering a wide range of path-length differences between both arms of the fiber interferometer that form the composite cavity. Experimental results are in good agreement with our model. In particular, the path length difference determines periodic frequency windows within which supermodes of the composite cavity are observed. Outside these windows, supermodes are not detectable. Our results show quite remarkably that, although the number of supermodes is reduced with respect to the simple cavity, no measurable reduction of the overall supermode noise is obtained, contrary to what has recently been suggested [1].**

***Index Terms*—Mode-locked lasers, noise, noise measurement, optical fiber lasers, optical pulse generation.**

I. Introduction

HARMONICALLY mode-locked erbium-doped fiber lasers are stable sources of picosecond pulse trains at gigahertz repetition rates that can be used in optical telecommunication or sensing applications. Due to the pigtails of fiber components, the length of the laser cavity can not be made shorter than a few meters. It can even be much longer, up to hundreds of meters, when nonlinear pulse shortening through propagation in a fiber is employed. As a result, the fundamental frequency of the laser cavity or its free spectral range (FSR) lies typically in the megahertz range. In order to obtain the gigahertz repetition rates needed for high-speed optical communications, it is necessary to drive the electrooptic modulator at a modulation frequency f_m, which is a very high harmonic N of the FSR: this is the principle of harmonic modelocking (i.e., $f_m = N \times FSR$). As a consequence of harmonic modelocking, the laser cavity has a large number of competing supermodes (i.e., N sets of synchronized cavity modes separated by the modulation frequency). Beating between supermodes is the main source of amplitude fluctuations of the mode-locked pulses, and can lead, in the worst case, to sporadic suppressions of pulses in the train (pulse drop-out). One solution is to make one supermode dominant and reduce all others.

Several techniques were proposed to reduce supermode beat noise. Harvey *et al.* used a high-finesse Fabry–Perot étalon filter, selecting only one supermode, but requiring a stabilization scheme [2]. In [3], dithering of the cavity length at kilohertz rates is thought to reduce spatial hole burning, resulting in a reduction of the unwanted supermodes, but this effect is not clearly understood. Other techniques demonstrated the use of intensity-dependent loss to reduce pulse amplitude fluctuations. Additive pulse limiting (APL) [4] relies on nonlinear polarization rotation in the presence of a polarizer. Because APL needs a non polarization-maintaining cavity, its implementation makes the laser sensitive to environmental perturbations. The combined effect of self-phase modulation (SPM) and optical filtering [5] is simple to implement, but several hundred meters of fiber are required to get a sufficient amount of SPM. Pulse-intensity limiting in the form of two-photon absorption by a semiconductor mirror is another technique which was recently demonstrated [6]. However, this technique does not rely on fiber-optics components. Finally, the composite cavity, whose principle was initially introduced for mode selection and wavelength tuning of CW fiber lasers [7], [8], was proposed to reduce supermode beat noise in harmonically mode-locked erbium-doped fiber lasers [1]. Among the aforementioned techniques, the composite cavity is the most simple and straightforward. However, a detailed model of the supermode noise structure with a composite cavity has not yet been presented.

In this paper, we consider the problem of supermode noise in a harmonically mode-locked fiber laser with a composite cavity (Fig. 1). We derive an analytical expression for the supermode structure and present experimental evidence supporting the theoretical results. Rather unexpectedly, our study demonstrates that the composite cavity does not lead to a significant reduction of the overall supermode noise, contrary to what has been suggested in [1].

II. Theory

A. Vernier Principle

In a ring fiber laser cavity of length L, the frequency spacing between cavity modes, i.e., free spectral range $\Delta\nu$, is given by

$$\Delta\nu = \frac{c}{n_e L} \tag{1}$$

where c is the light velocity in vacuum and n_e is the effective refractive index of fiber (~1.5 in silica). As a starting point,

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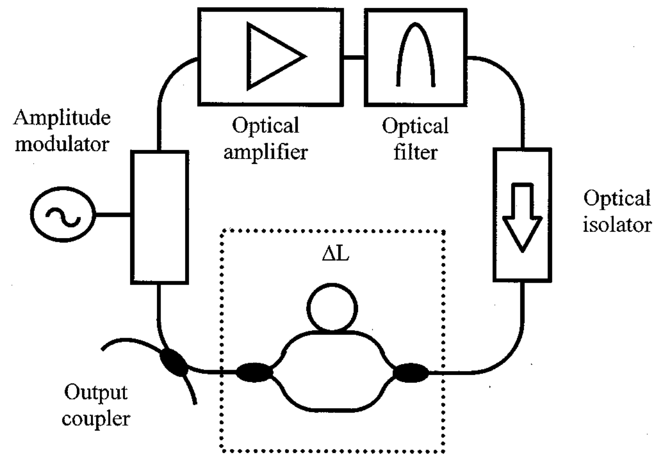

Fig. 1. General structure of an actively and harmonically mode-locked fiber ring laser with a composite cavity. This structure is obtained by inserting an all-fiber Mach–Zehnder interferometer having path length difference ΔL (dashed frame) into a ring laser cavity The direction of propagation of intracavity light is imposed by the optical isolator.

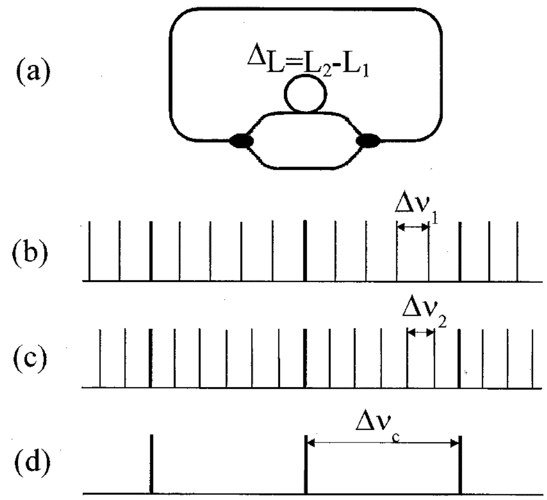

Fig. 2. Schematic view of the: (a) composite cavity consisting of two sub-cavities of lengths L_1 and L_2; (b) modes of a single cavity of length L_1; (c) modes of a single cavity of length L_2; and (d) modes of the composite cavity. The mode spacing, or free spectral range, of the cavity is noted $\Delta\nu$.

assume a ring laser of length L_1 and free spectral range $\Delta\nu_1$. The principle of the composite cavity is to virtually shorten the cavity, i.e., to increase its free spectral range, in order to reduce the order of harmonic modelocking and, thereby, the number of supermodes for a given modulation frequency. The composite cavity [Fig. 2(a)] is formed by coupling the first cavity to a second cavity of length $L_2 = L_1 + \Delta L$ ($\Delta\nu_2 < \Delta\nu_1$ if $\Delta L > 0$). In the scheme of Fig. 2(a), this is realized by introducing two different paths for the light in the ring. The two fiber sections with path length difference ΔL inserted between 50/50 couplers form a Mach–Zehnder fiber interferometer. The composite cavity can be regarded as the superposition of two simple cavities of lengths L_1 and L_2. In the following, we assume that the path length difference ΔL is much smaller than the absolute lengths, so that $L_1 \approx L_2$. As a consequence, $\Delta\nu_1$ and $\Delta\nu_2$ are close to each other but still different. Suppose there exists a lowest common multiple $\Delta\nu_c$ of $\Delta\nu_1$ and $\Delta\nu_2$. In this case, particular modes that are separated by $\Delta\nu_c$ match each other's frequency [Fig. 2(b) and (c)]: this is the Vernier principle [1]. Assuming that only these modes that are common to both simple cavities can exist [Fig. 2(d)], $\Delta\nu_c$ is the effective free spectral range of the composite cavity. It is given by

$$\Delta\nu_c = N_1 \times \Delta\nu_1 = N_2 \times \Delta\nu_2 \tag{2}$$

with N_1 and N_2 being the smallest integers verifying (2) (N_1 and N_2 have no common divisors).

This intuitive description of the composite cavity shows that $\Delta\nu_c$ is determined solely by ΔL once L_1 (or L_2) is given. As the goal of the composite cavity is to drastically decrease the order of harmonic modelocking N, and thereby the number of supermodes, ΔL has to be chosen so as to make $\Delta\nu_c$ very large compared to $\Delta\nu_1$ and $\Delta\nu_2$, but of course not larger than the desired modulation frequency f_m. In the limit case where $\Delta\nu_c = f_m$, modelocking is virtually achieved at order $N = 1$, i.e., as if it were fundamental modelocking. In all cases, the large value of $\Delta\nu_c$ corresponds to a virtually short cavity, usually impossible to realize in practice.

B. Model

According to the Vernier principle (Fig. 2), the resonant modes of the composite cavity consist of a comb of frequencies separated by $\Delta\nu_c$. If the composite cavity laser is harmonically mode-locked, supermodes are expected at frequencies that match exactly the comb. In our experiments, however, supermodes were also observed at frequencies that were shifted from the comb, with intensities that decrease as the shift increases (see Section III). The explanation of these observations clearly requires a more detailed analysis than the simple description relying on the Vernier principle. The model we propose here is based on the transfer function of the composite cavity, assuming that gain compensates for the loss in one roundtrip. This model will highlight both the resonance condition in the ring and the interference condition in the fiber interferometer.

In order to calculate the transfer function of the composite cavity, let us virtually open the laser ring at any point outside the fiber interferometer and take one end as input and the other as output of the system (Fig. 3, left). This system can be considered as the superposition of two subcavities of lengths $L_1 = l_o + l_1$ and $L_2 = l_o + l_2$; one half of the input electric field E_{in} travels along L_1, and the other half along L_2 (Fig. 3, right). Assuming that gain compensates for the loss of the system, the output electric field E_{out} is expressed by

$$\begin{aligned} E_{\text{out}}(t) &= E_{\text{out1}}(t) + E_{\text{out2}}(t) \\ &= \frac{1}{2} E_{\text{in}}\left(t - \frac{n_e L_1}{c}\right) + \frac{1}{2} E_{\text{in}}\left(t - \frac{n_e L_2}{c}\right). \end{aligned} \tag{3}$$

Considering a monochromatic input field $E_{\text{in}} = E_i \times \exp(j\omega t)$, where $\omega = 2\pi \times f$ is the frequency (in radians/second) and $E_i > 0$, the output field E_{out} has the form $E_{\text{out}} = E_o \times \exp[j(\omega t - \phi)]$ (with $E_o > 0$), for which (3) yields

$$E_{\text{out}}(t) = E_i \cos\left(\omega \frac{n_e \Delta L}{2c}\right) \times e^{j\omega(t - n_e L_m/c)} \tag{4}$$

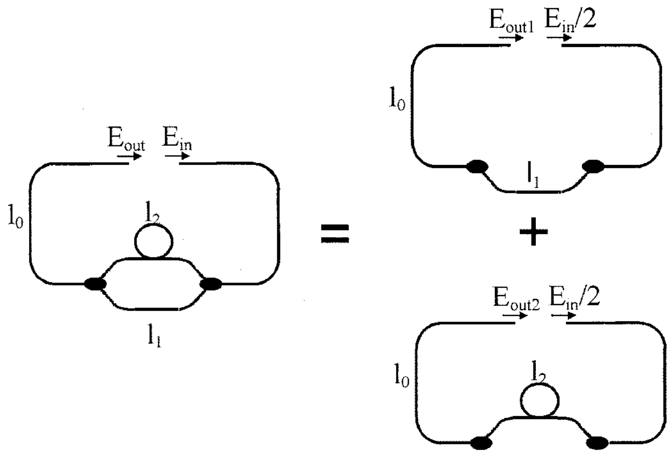

Fig. 3. In order to calculate the transfer function of the composite cavity (left), one considers the composite cavity as the superposition of two subcavities (right) of lengths $L_1 = l_o + l_1$ and $L_2 = l_o + l_2$.

where $\Delta L = L_2 - L_1 = l_2 - l_1$ and $L_m = (L_1 + L_2)/2$. From (4), we can derive the transmittance of the system $T_f(\omega)$

$$T_f(\omega) = \left(\frac{E_o}{E_i}\right)^2 = \frac{1}{2}\left[1 + \cos\left(\omega \frac{n_e \Delta L}{c}\right)\right] = \frac{1}{2}\left[1 + \cos\left(2\pi \frac{f}{\Delta\nu_f}\right)\right] \quad (5)$$

where $\Delta\nu_f$ is given by (1) with L replaced by ΔL. Since $\Delta L \ll L_1, L_2$, we have $\Delta\nu_f \gg \Delta\nu_1, \Delta\nu_2$. We note that $T_f(f)$ is also the expression of the transmittance of the Mach–Zehnder interferometer, which is actually a low-finesse filter (as far as the *amplitude* of the electric field is considered, the system reduces to the interferometer since it is assumed to have no net gain or loss). The phase of E_{out} is easily calculated from (4)

$$\phi = \frac{\omega n_e L_m}{c}, \quad \text{if } 2m\pi - \frac{\pi}{2} \le \omega \frac{n_e \Delta L}{2c} \le 2m\pi + \frac{\pi}{2}$$
$$\phi = \frac{\omega n_e L_m}{c} + \pi, \quad \text{if } 2m\pi + \frac{\pi}{2} < \omega \frac{n_e \Delta L}{2c} < 2m\pi + \frac{3\pi}{2}. \quad (6)$$

Note that the π phase jumps appearing in the expression of ϕ are due to the presence of the cosine function in (4). Therefore, according to (6), the frequencies of the composite cavity modes, i.e., the frequencies at which the phase difference ϕ between E_{in} and E_{out} is an integral multiple of 2π, are given by

$$f = q\Delta\nu_m, \quad \text{if } \left(2m - \tfrac{1}{2}\right)\Delta\nu_f \le f \le \left(2m + \tfrac{1}{2}\right)\Delta\nu_f$$
$$f = \left(q - \tfrac{1}{2}\right)\Delta\nu_m, \quad \text{if } \left(2m + \tfrac{1}{2}\right)\Delta\nu_f < f < \left(2m + \tfrac{3}{2}\right)\Delta\nu_f \quad (7)$$

where q, m are integers, and $\Delta\nu_m = c/(n_e \times L_m)$. $\Delta\nu_m$ can be expressed as a function of $\Delta\nu_1$ and $\Delta\nu_2$ as follows:

$$\Delta\nu_m = \frac{2\Delta\nu_1 \Delta\nu_2}{\Delta\nu_1 + \Delta\nu_2}. \quad (8)$$

The transmittance of the composite cavity $T_f(f)$ is represented in Fig. 4 as a function of the normalized frequency $f/\Delta\nu_m$, for different values of the parameter $\Delta\nu_f/\Delta\nu_m = L_m/\Delta L$. Modes defined by (7) are also displayed in Fig. 4 as vertical lines. Successive modes are separated by $\Delta\nu_m = 2 \times c/[n_e \times (L_1 + L_2)] = c/(n_e \times L_m)$, except around the zeros of $T_f(f)$, where a frequency shift of $\Delta\nu_m/2$ occurs [cf. (7)]. It means that if the modes in a transmission window between two zeros of $T_f(f)$ have frequencies that are integer multiples of $\Delta\nu_m$, the modes in the next window have frequencies that are half-integer multiples of $\Delta\nu_m$, and conversely. Therefore, quite remarkably, the modes of a composite cavity are made of successive regular combs that are shifted from one to another by $\Delta\nu_m/2$. Because we have assumed that $\Delta L \ll L_1, L_2$ and since $\Delta\nu_2 < \Delta\nu_m < \Delta\nu_1$ by definition, we have $\Delta\nu_m \cong \Delta\nu_1, \Delta\nu_2$. The parameter $\Delta\nu_m$ is related to the resonant conditions of the two ring subcavities of lengths L_1 and L_2. On the other hand, $\Delta\nu_f$, which is the period of $T_f(f)$, is related to the constraint imposed by the composite cavity, involving path length difference ΔL. Therefore $\Delta\nu_f/\Delta\nu_m = L_m/\Delta L \gg 1$. In Fig. 4, $\Delta\nu_f/\Delta\nu_m$ was chosen relatively small, for the purpose of clarity.

We shall show now that our model is in agreement with the intuitive description relying on the Vernier principle. From classical laser theory, the steady-state gain is saturated at a value corresponding to minimal loss, so that only the modes corresponding to the maximum of the transmittance $T_f(f)$ exist in steady state (bold lines in Fig. 4). Depending on the ratio between $\Delta\nu_f$ and $\Delta\nu_m$, the separation between these modes, i.e., the FSR of the composite cavity, denoted $\Delta\nu_c$, will be different. In Fig. 4(a), modes exist at all maxima of $T_f(f)$. This situation occurs when the following condition is fulfilled

$$\frac{\Delta\nu_f}{\Delta\nu_m} = \frac{L_m}{\Delta L} = Q + \frac{1}{2} \quad (9)$$

where Q is an integer, and the term 1/2 takes into account the aforementioned frequency shift of $\Delta\nu_m/2$ at the zeros of $T_f(f)$. If (9) is satisfied, then $\Delta\nu_c = \Delta\nu_f$, i.e., the FSR of the composite cavity is equal to the period of the transmittance, as depicted in Fig. 4(a). Let us show that this result is in agreement with that obtained by applying the Vernier principle. Using (9) and definitions of $\Delta\nu_1$ and $\Delta\nu_2$ [see (1)], it can be shown that

$$\Delta\nu_f = Q\Delta\nu_1 = (Q+1)\Delta\nu_2 = \Delta\nu_c. \quad (10)$$

Equation (10) shows that $\Delta\nu_f$ is equal to the lowest common multiple $\Delta\nu_c$ of $\Delta\nu_1$ and $\Delta\nu_2$. Indeed, (10) is nothing but (2) with $N_1 = Q$ and $N_2 = Q + 1$.

In Fig. 4(b) and (c), modes do not match all the maxima of $T_f(f)$. This situation occurs when the following condition is fulfilled:

$$\frac{\Delta\nu_f}{\Delta\nu_m} = \frac{L_m}{\Delta L} = Q + \frac{1}{2} + \frac{R}{K} \quad (11)$$

where Q, R, K are integers ($R \neq 0$, $K > 0$, $|R| < K$ and R, K have no common divisors). Equation (11) imposes that modes exist only at one maximum of $T_f(f)$ every K maxima [$K = 2$ in Fig. 4(b) and $K = 3$ in Fig. 4(c)]. If (11) is satisfied, then $\Delta\nu_c = K \times \Delta\nu_f$, i.e., the FSR of the composite cavity is an integer multiple of the period of the transmittance. Again, using (11) and (1)

$$K \times \Delta\nu_f = (QK + R)\Delta\nu_1 = ((Q+1)K + R)\Delta\nu_2 = \Delta\nu_c. \quad (12)$$

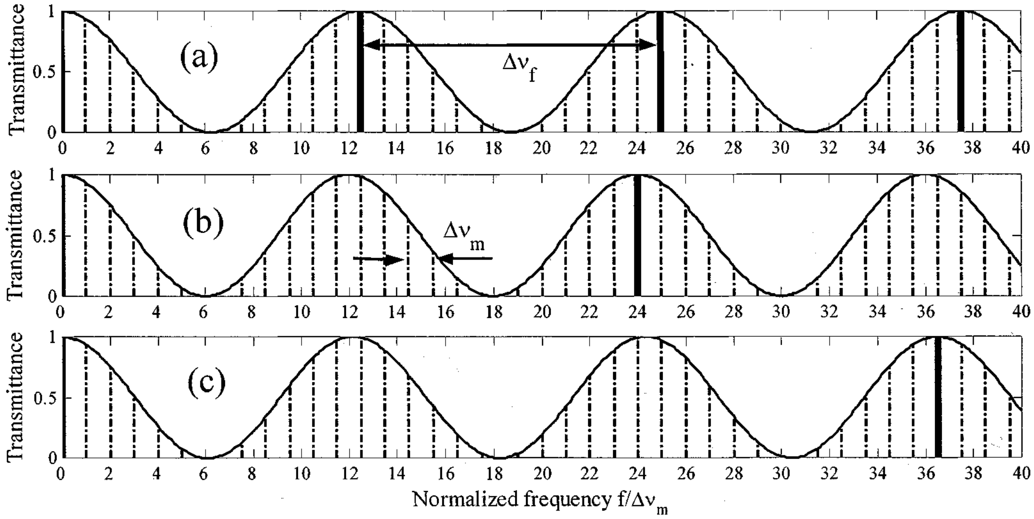

Fig. 4. Transmittance of the composite cavity for different values of the parameter $\Delta\nu_f/\Delta\nu_m = L_m/\Delta L =$: (a) 12.5, (b) 12.0, and (c) 12.17. Vertical lines are drawn at frequencies defined by (7). Bold (dashed) lines correspond to frequencies at which transmittance is (less than) unity.

Equation (12) shows that $K \times \Delta\nu_f$ is the lowest common multiple $\Delta\nu_c$ of $\Delta\nu_1$ and $\Delta\nu_2$ since R and K have no common divisors. Indeed, (12) is nothing but (2) with $N_1 = Q \times K + R$ and $N_2 = (Q+1) \times K + R$. The FSR of the composite cavity obtained by our model is again in agreement with that obtained by applying the Vernier principle. Note that the case of Fig. 4(a) described above simply corresponds to $K = 1$ and $R = 0$.

In the case where the ratio between L_m and ΔL, i.e., the ratio between $\Delta\nu_f$ and $\Delta\nu_m$, is not rational, no integers Q, R, K can be found that satisfy (11). As a consequence, considering that one mode matches maximal transmittance, no other mode can be found at another maxima of the transmittance, contrary to Fig. 4. This case thus corresponds to an infinite value of the FSR of the composite cavity, $\Delta\nu_c = \infty$. Letting $\Delta\nu_c = K \times \Delta\nu_f$, and remembering that $\Delta\nu_f$ is finite (and is imposed physically by ΔL), we find that, in this case, $K = \infty$.

In summary, we have shown that when it is finite, the FSR of the composite cavity is an integer multiple of the period of the transmittance $\Delta\nu_c = K \times \Delta\nu_f$. The value of K is determined by L_m and ΔL, as shown by (11), or equivalently by the values of L_1 and ΔL. Note that the value of K, hence $\Delta\nu_c$, has intrinsically no superior limit except that the FSR must be lower than (or equal to) the modulation frequency f_m for harmonic (fundamental) modelocking. Compared to the case $K = 1$, the case $K > 1$ seems attractive because a large FSR $\Delta\nu_c$ can be achieved with a smaller $\Delta\nu_f$, i.e., with a larger ΔL, which is easier to set in practice. Unfortunately, for a practical laser, we will show in the following that the case $K > 1$ is not relevant.

In a practical laser, the ratio $L_m/\Delta L$ is often much higher than in the case of Fig. 4, typically on the order of several hundreds or even thousands. Fig. 5 plots the transmittance $T_f(f)$ computed for different values of $L_m/\Delta L$ corresponding to the actual lengths used for experiments. For each curve, the value of L_m is approximately the same but values of ΔL are different. The frequency range is limited to a few multiples of $\Delta\nu_m$ on either side of the maximum of transmittance. From the analysis of Fig. 5, it is clear that the difference in transmittance (or loss) between the mode centered at maximum and adjacent modes is very small and decreases with increasing $L_m/\Delta L$. In practice, a loss difference between the modes lower than, say 10^{-2}, is negligible. This means that, in contrast to the conclusions of [1] based on the Vernier principle, not only can the mode that matches the maximum exist, but also adjacent modes whose number increases with increasing values of $L_m/\Delta L$. Further, modes can also exist about maxima of $T_f(f)$ that do not match perfectly the mode frequencies [Fig. 4(b) and (c)]. As a consequence, whatever the value of K, modes will be found about all maxima of transmittance, as in the case $K = 1$, although not at the exact frequency values corresponding to these maxima. For this reason, the case $K > 1$ is not relevant in practice. Moreover, environmental perturbations, such as temperature variations and vibrations, induce fluctuations of the cavity length and thereby

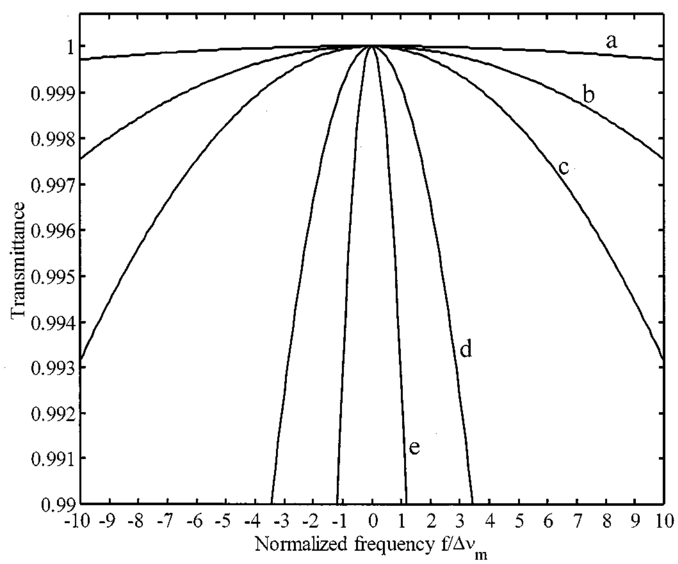

Fig. 5. Transmittance of the composite cavity in a narrow frequency range around the maximum transmittance for different values of $L_m/\Delta L$ used in the experiments. (a)–(e): $L_m/\Delta L$ = 1833, 635, 379, 108, and 37. They correspond, respectively, to N = 1, 3, 5, 18, and 50, where $N = f_m/\Delta\nu_f$ and f_m is the modulation frequency (see Section III).

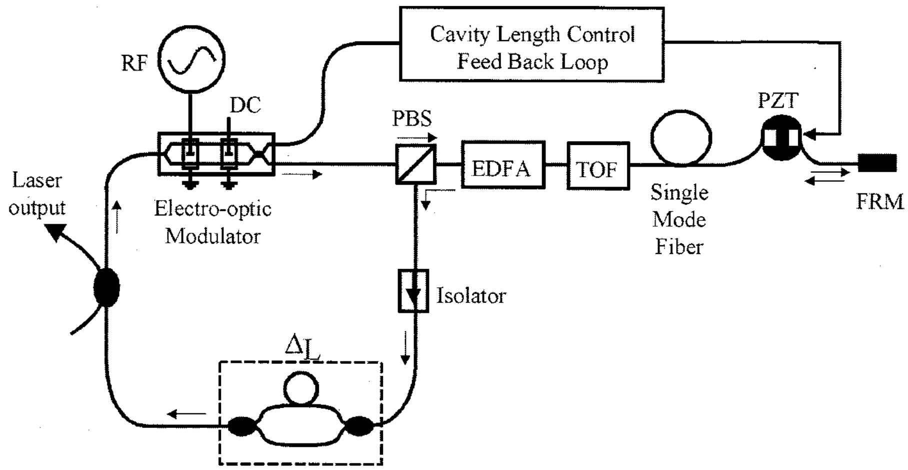

Fig. 6. Harmonically mode-locked erbium-doped fiber laser with composite cavity. EDFA: erbium-doped fiber amplifier. TOF: tunable optical filter. PBS: polarization beam splitter. FRM: Faraday rotation mirror. PZT: piezoelectric transducer. The basic laser has a sigma cavity. The composite cavity is implemented by inserting a Mach–Zehnder fiber interferometer in the ring (dashed box). For environmental stability of the laser, the cavity length is controlled through a piezoelectric transducer on which single-mode fiber is wound. Principle and implementation of the stabilization feedback loop are explained in [9].

fluctuations of the free spectral range. It means that $\Delta\nu_f$ and $\Delta\nu_m$ can fluctuate during experiments. Because $\Delta\nu_m$ depends on both L and ΔL while $\Delta\nu_f$ depends solely on ΔL, the relative positions of the modes with respect to the maxima of the transmittance can fluctuate slightly in time. This is an additional cause of the appearance of modes in the vicinity of the transmittance maxima.

III. Experiment

For experiments, we use an harmonically mode-locked erbium-doped fiber laser (Fig. 6). The basic laser has a sigma cavity [10] formed by a polarization-maintaining ring (Fig. 6, left) and a double-pass single-mode section (Fig. 6, right) terminated by a Faraday rotation mirror. Thanks to the Faraday mirror, polarization fluctuations in the single-mode section are cancelled and the sigma cavity is functionally equivalent to a unidirectional polarization-maintaining ring. The polarization-maintaining property of the laser is very important, as it enhances considerably the stability of the pulse generation. The composite cavity is implemented by inserting in the ring a Mach–Zehnder fiber interferometer made of two 50/50 polarization-maintaining couplers and polarization-maintaining fiber pigtails (dashed box in Fig. 6). The path length difference ΔL of the interferometer is set to the desired value by cutting pigtails at appropriate lengths.

In our experiments, we measure the supermode noise of the laser with a composite cavity having different free spectral ranges, i.e., different values of ΔL. For comparison, we also measure the supermode noise of the laser with a single cavity, which is realized by disconnecting one arm of the fiber interferometer. The pulse train is detected using a 20-GHz photodiode connected to both a sampling oscilloscope (20-GHz bandwidth) and an electrical spectrum analyzer (22-GHz bandwidth). The sampling oscilloscope is set to infinite persistence mode. Pulse amplitude fluctuation results in scattering of the pulse peak on the trace. This measurement, however, does not allow to separate supermode noise from other contributions to amplitude noise (e.g., relaxation oscillations). In order to quantify the supermode noise, we measure the radio-frequency (RF) spectra using an electrical spectrum analyzer. We estimate the suppression of supermodes with respect to the carrier by calculating the signal-to-noise ratio (SNR), defined by

$$\mathrm{SNR} = -10\log\left(\frac{\frac{1}{M}\sum P_{m,i}}{P_c}\right) \tag{13}$$

where P_c is the power of frequency component at f_m (carrier), $M = f_m/\Delta\nu_m$, $P_{m,i}$ is the power of each mode i that is measured individually, and the summation is extended to all modes comprised between f_m and $2\times f_m$. The numerator in the right-hand side of (13) is the average power level of the supermode noise. This definition of SNR is adapted to the composite cavity, in which the power level of supermodes is not uniform with respect to frequency. Equation (13) is a generalization of the usual definition of supermode suppression ratio in a single cavity, in which the power level of supermodes is uniform with respect to frequency. The value of the SNR is expressed in decibels, with higher SNR meaning that supermode noise is more suppressed with respect to carrier.

The basic FSR of the laser is $\Delta\nu_1 \approx 1.6$ MHz ($L_1 \approx 125$ m). The laser is harmonically mode-locked at $f_m \approx 3$ GHz. Different values of path length difference ΔL are chosen to satisfy $\Delta\nu_f = c/(n_e \times \Delta L) = f_m/N$ with $N = 1$, 3, 5, 18, 50, 194, and 426, respectively. Note that the case $N = 1$ corresponds to virtually fundamental modelocking (cf. Section II-A). The modulation frequency lying in the gigahertz range, the values of ΔL range from 6.8 cm ($N = 1$) to more than 28 m ($N = 426$). In practice, f_m is finely tuned in order to match at best the desired multiple of $\Delta\nu_f$, physically imposed by ΔL. In the following, the single-cavity case is noted $N = 0$, i.e., equivalent to $\Delta L = 0$. The optical filter bandwidth was 3 nm (full-width at half-maximum). The pulse duration was lying in the range 10–15 ps in all cases, and showed no significant dependence on the value of ΔL.

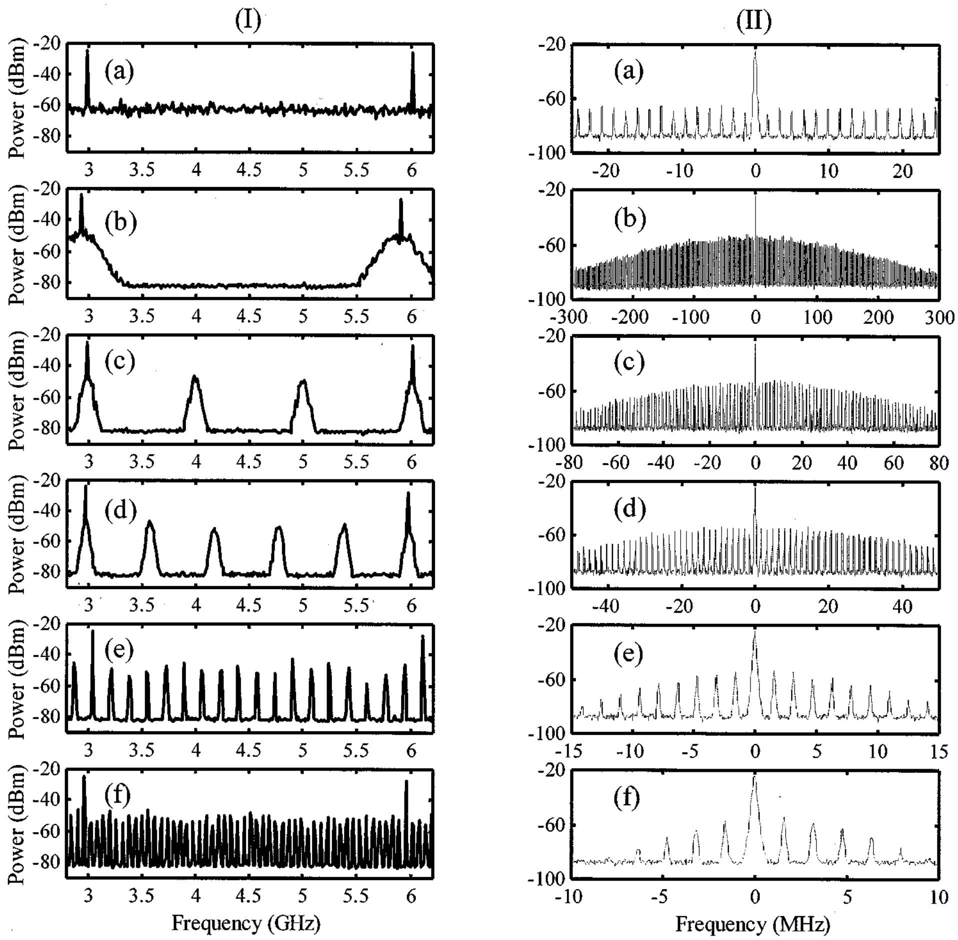

Fig. 7. RF spectra of the detected pulse train from harmonically mode-locked laser with (a) simple cavity and (b)–(f) composite cavity having different values of ΔL. Modulation frequency f_m is about 3 GHz in all cases. Values of ΔL for the composite cavity are chosen to satisfy $\Delta\nu_f = c/(n_e \times \Delta L) = f_m/N$ with $N = 1, 3, 5$, 18 and 50 from top to bottom respectively. (I) On the left panels, spectra are measured between f_m and $2 \times f_m$ with a constant resolution bandwidth of 300 kHz. (II) On the right panels, spectra are measured on a short span around f_m, using a constant resolution bandwidth of 100 kHz. In order to resolve all individual supermodes, some of the figures (II) are obtained by concatenation of several spectra measured using shorter spans.

For the cases $N = 0$, 1, 3, 5, 18, and 50, the RF spectra of the detected pulse train measured between the first and second harmonics of f_m and measured on a short span around f_m are shown in Fig. 7 (I) and (II), respectively. For convenience, the cases $N = 194$ and 426 are shown separately on a short span in Fig. 8 as $\Delta\nu_f$ is much smaller in these cases. Some important observations can be made from the analysis of Fig. 7 and Fig. 8. In the single-cavity case [Fig. 7(a)], as expected, all supermodes are present between f_m and $2 \times f_m$, and their power level is roughly constant over the whole frequency range (in Fig. 7 (I), (a), the constant level observed between f_m and $2\times f_m$ is that of the supermodes, which are not resolved individually. It is higher than the noise level of the measurement setup, as is clearly visible in Fig. 7(II), (a) with a higher resolution). In the composite-cavity cases, groups of supermodes are observed within periodic frequency windows, some of them being centered at the peaks of the leading supermode (at integer multiples of f_m) and separated by $\Delta\nu_f$ (Fig. 7(I), (b)–(f) and Fig. 8). Within each window, the power of a supermode decreases as its frequency offset from the center of the window increases (Fig. 7(II), (b)–(f) and Fig. 8). The width of these windows, i.e., of the frequency domains where bunches of modes are observed, is inversely proportional to their number (N) between f_m and $2 \times f_m$. Between the windows, the power level is equal to the noise floor of the detection setup, which means that the supermodes—if they exist—have extremely small intensities at these frequencies. The *number* of supermodes that are detectable is, therefore, reduced in comparison with the single cavity. However, we note that the *maximum* power level of supermodes of the composite cavity is about the same in all cases although it is *higher* than the *constant* power level of supermodes of the single cavity (cf. Fig. 7). The values of SNR obtained by (13) are calculated from RF spectra and found to be similar in all cases: $\mathrm{SNR} = 42.7$, 42.9, 41.5, 41.7, 41.3, 40.7, 37.5, and 40.4 dB (standard deviation: 1.6 dB) for $N = 0$, 1, 3, 5, 18, 50, 194, and 426, respectively. Note that, although the power of the leading supermode is in all cases about 40 dB above the average power of other supermodes, supermode beat noise is large, given the large number of unwanted supermodes ($M = f_m/\Delta\nu_m \cong 2000$). Finally, the oscilloscope traces of the pulses with infinite persistence mode show the same amount of scattering of the pulse peak for both single and composite cavities.

In the cases $N = 194$ and 426 (Fig. 8), ΔL is large (13 m and 28.4 m, respectively). As we shall see below, an interesting consequence of the large ΔL is that it allows us to evaluate parameters of our model that are not accessible in the other cases due to

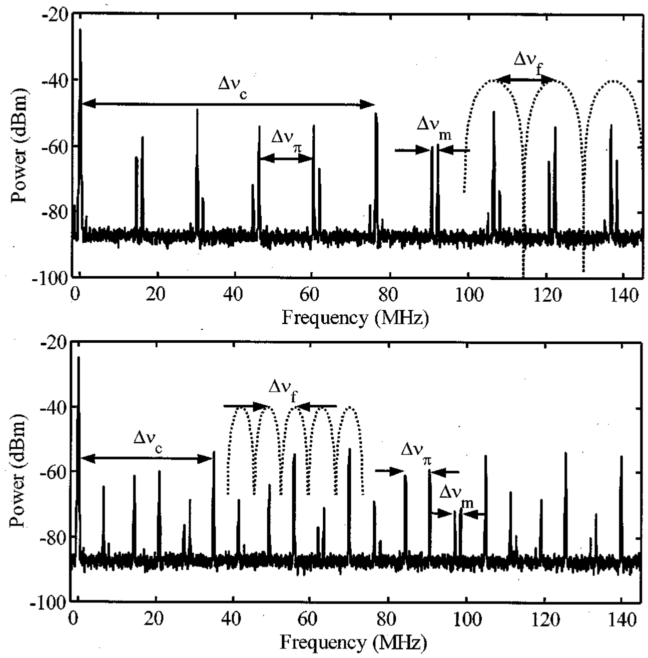

Fig. 8. RF spectra of the detected pulse train from harmonically mode-locked laser with composite cavity having large values of ΔL. Modulation frequency f_m is about 3 GHz in both cases and values of ΔL satisfy $\Delta\nu_f = c/(n_e \times \Delta L) = f_m/N$ with $N = 194$ and 426 from top to bottom respectively. The first (highest) peak on the left of the figures corresponds to the modulation frequency f_m. Resolution bandwidth is set to 100 kHz. Dotted curves reproduce the shape of the transmittance, as a guide for the eyes (arbitrary maxima).

limited resolution of spectral measurements (cf. Section II-B). In the present cases, as L_1 and L_2 are significantly different, $\Delta\nu_m$ can be easily distinguished from $\Delta\nu_1$ and $\Delta\nu_2$ in spectral measurements. With the RF spectrum analyzer, we measure the free spectral ranges $\Delta\nu_1$ and $\Delta\nu_2$ of the single cavities (by disconnecting successively the arms of the interferometer) and $\Delta\nu_m$ of the composite cavity. We find that the measured value of $\Delta\nu_m$ is identical to the one calculated from (8), given the measured values of $\Delta\nu_1$ and $\Delta\nu_2$. The choice of a large ΔL also implies that $\Delta\nu_f$ is only a few times larger than $\Delta\nu_m$, with the consequence that only one, two, or at most three modes are observed in each frequency window, and their number and level are different depending on whether they match the maxima of transmittance $T_f(f)$ or not (as shown in Fig. 8). This makes it possible to measure two important parameters. The first is $\Delta\nu_\pi$, the frequency interval between any two modes chosen in consecutive windows (Fig. 8). As several modes appear in each window, it comes from the above that $\Delta\nu_\pi$ is defined to within a few times $\Delta\nu_m$. Note that the interval $\Delta\nu_\pi$ between modes is close to—although different from—$\Delta\nu_f$, which is the frequency interval between successive maxima of transmittance, where modes are not necessarily present. In Fig. 8, we find in all cases that $\Delta\nu_\pi = (Q+1/2)\times\Delta\nu_m$, where Q is an integer, a result which is in agreement with the frequency shift of $\Delta\nu_m/2$ at each minimum of the transmittance predicted by (7). The second parameter is $\Delta\nu_c$, the frequency interval between the consecutive modes that match exactly the maxima of transmittance. We find for both $N = 194$ and 426 that $\Delta\nu_c = 5 \times \Delta\nu_f$ (i.e., $K = 5$, cf. Section II-B). In Fig. 8, the period $\Delta\nu_c$ is clearly observable. However, it is important to note that modes that do not match the maxima of transmittance are far from being suppressed in practice, even for $N = 426$. The results presented here are summarized in Table I.

TABLE I

PARAMETERS OF THE COMPOSITE CAVITY MEASURED IN THE CASES $N = 194$ AND $N = 426$. THE TABLE ALSO INCLUDES VALUES OF $\Delta\nu_m$ CALCULATED FROM THE MEASURED QUANTITIES $\Delta\nu_1$ AND $\Delta\nu_2$ USING (8). INTEGERS Q, R AND K ARE OBTAINED FROM VALUES OF $\Delta\nu_f/\Delta\nu_m$ USING (11). K CAN ALSO BE OBTAINED BY THE RATIO $\Delta\nu_c/\Delta\nu_f$

N	194	426
f_m (GHz)	2.987	3.005
$\Delta\nu_1$ (MHz)	1.609	1.609
$\Delta\nu_2$ (MHz)	1.450	1.307
$\Delta\nu_m$ (measured) (MHz)	1.525	1.450
$\Delta\nu_m$ (calculated) (MHz)	1.525	1.442
$\Delta\nu_\pi / \Delta\nu_m$	9.48; 10.49	4.50; 5.49
$\Delta\nu_f$ (MHz)	15.40	7.054
$\Delta\nu_c$ (MHz)	76.98	35.27
$\Delta\nu_f / \Delta\nu_m$	~10.1 = 9 + 1/2 + 3/5	~4.9 = 4 + 1/2 + 2/5
Q	9	4
R	3	2
K	5	5

IV. DISCUSSION

The supermodes having the highest power are observed around frequencies that are integral multiples of $\Delta\nu_f$ [Fig. 7(I)]. This is consistent with our model of the composite cavity (Section II-B), the frequencies of these highest supermodes corresponding to the maxima of the transmittance $T_f(f)$ (Fig. 4). Moreover, the fact that the intensity of supermodes decreases as their frequency offset from these maxima increases [Fig. 7(II)] is also consistent with the model (Fig. 5). A main result of the experiments is that supermodes of the composite cavity are only detectable in periodic frequency windows separated by $\Delta\nu_f$. As ΔL increases, the number of such windows increases, but their width—and therefore the number of supermodes they contain—decreases, while the power level of the higher-power unwanted supermodes remains unchanged. Therefore, as ΔL is changed, the average power level of the supermode noise remains approximately the same. This is not a surprising result if we remember that the finesse of the fiber interferometer is constant.

Extending our experimental investigation to the case of large values of ΔL, we show that the measured value of $\Delta\nu_m$ is in agreement with our model. In addition, we observe the $\Delta\nu_m/2$ frequency shift of the modes at the minima of the transmittance predicted by our model. Finally, we measure $\Delta\nu_c$ in cases where $K \neq 1$ ($\Delta\nu_c \neq \Delta\nu_f$) and observe that modes which do not match the maxima of the transmittance also exist, again as predicted by the model.

V. CONCLUSION

We have studied, both theoretically and experimentally, the supermode noise of harmonically mode-locked erbium-doped fiber lasers with a composite cavity. On the one hand, we have proposed a model of the composite cavity that is able to determine both the frequencies encountering minimal loss in the cavity and the frequencies of the cavity modes. The model shows that the modes of the composite cavity have this peculiar property that their frequencies undergo a periodic jump at every zero of the cavity transmittance. On the other hand, an exhaustive experimental study of the composite cavity was carried out, covering a wide range of path length difference ΔL. These results were found to be in good agreement with the above mentioned theoretical results obtained with our model. The experimental results also showed that in RF spectra of the detected pulse train, supermodes of the composite cavity are only detectable in periodic frequency windows in which supermode noise is concentrated. In accordance with our model, the period and width of these windows were found to be inversely proportional to the path length difference in the fiber interferometer, while their amplitude is about constant. This is consistent with the experimental evidence that the average power level of the supermode noise does not depend on the path length difference. Finally, our results show, quite remarkably that, in comparison with a single cavity, the number of supermodes is reduced although no measurable reduction of the overall supermode noise is obtained, contrary to what has recently been suggested [1].

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