Multibody dynamics
Multibody simulation of an industrial robot in contact with a deformable environment with friction

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Overview

1. Introduction
   - Considered case

2. Robot model
   - Multibody model
   - Model of the controllers

3. Environment model
   - Environment model
   - Friction model

4. Integration and results
   - No contact
   - Contact
   - Integration of the differential equations
   - Considered cases
   - Some results

5. Conclusions

Multibody dynamics
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Multibody dynamics
Considered case

- a 6-DOF robot in contact with a deformable environment with friction.
Purpose:

- gain a better understanding of the dynamic behaviour.
- compare how these results differ from experimental results (e.g. due to flexibility in the transmissions).
- for future (simulation) purposes: torque control, active end-effector, ...
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Multibody dynamics
Multibody model: first three axes

Model of the first three axes:

- inverse dynamic model:
  \[ \tau = M(q) \cdot \ddot{q} + C(q, \dot{q}) \cdot \dot{q} + G(q) + F(\dot{q}) \]
  - \( \tau \) the joint torques, \( q \) the joint angles, . . .
  - \( M(q) \) the joint angle dependent inertia matrix
  - \( C(q, \dot{q}) \) the centrifugal and Coriolis forces
  - \( G(q) \) gravitation
  - \( F(\dot{q}) \) friction

- can be rewritten as \( \tau = \phi(q, \dot{q}, \ddot{q}) \cdot \theta \), a model that is linear in the parameters \( \theta \) (inertia, friction, . . .).

- inverse model generated with Robotran
- based on cooperation of PMA/UCL
Multibody model: first three axes
Multibody model: axes four to six

Model of axes four to six:
- simple inverse dynamic model: $\tau = M \cdot \ddot{q} + F(\dot{q})$,
  - constant inertia matrix $M$
  - $F(\dot{q})$ accounts for friction
- parameter identification by C. Ganseman (1998)
Controllers

- velocity controllers: lag controllers \((K_l \cdot \frac{\tau_1 \cdot s + 1}{\tau_2 \cdot s + 1})\)
- position controllers: proportional controllers
- yields differential equations: \(\ddot{\tau} = S(q_d, \dot{q}_d, \ddot{q}_d, q, \dot{q}, \ddot{q}, \tau)\)
  - \(q_d\) is the input or "driving variable"

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General contact dynamics model


- how to choose the "contact dynamics" parameters?
- requires small stepsize (stability issues if stepsize too large or not enough damping).
- quite easy to integrate and switch between contact and no contact.
Less general contact dynamics model


- How to choose the "contact dynamics" parameters?
- Requires special attention when switching between contact and no contact (infinite contact force, calculate correct initial conditions).
Simplified contact dynamics model

- Environment is flat and located at $x = x_0 = 1\ m$
- Nonlinear environment model: $F_e = K_e \cdot \delta^n + \lambda_e \cdot \dot{\delta} \cdot \delta^n$
  - $\delta$ is the environment deformation
  - No force jumps (as opposed to linear model $c \cdot \dot{\delta}$)
  - For $n = 3/2$ and $\dot{\delta} = 0$ Hertz model
- No arbitrarily chosen parameters.
- Easier to switch between contact and no contact.
- Straightforward to integrate.
Coulomb and viscous friction

\[ F_f = F_n \cdot (\mu \cdot \text{sign}(v_{r,ee}) + c \cdot v_{r,ee}). \]

- \( v_{r,ee} \) is the end-effector velocity projected onto a plane parallel to the environment surface.
- the \( \text{sign}(v_{r,ee}) \) is replaced by the sigmoid function \( \sigma \) so as to avoid switching.
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In the case of no contact:

\[ M(q) \cdot \ddot{q} = \tau - C(q, \dot{q}) \cdot \dot{q} - G(q) - F(\dot{q}) \]

\[ 0 = K_e \cdot \delta^n + \lambda_e \cdot \dot{\delta} \cdot \delta^n \]

\[ \dot{\tau} = S(q_d, \dot{q}_d, \ddot{q}_d, q, \dot{q}, \ddot{q}, \tau) \]

(6 x 2) + 1 + 6 equations

\( \delta \) and \( q \) evolve independently.

switching conditions: if \( x_{ee} - x_0 > \delta \), then contact is made.
In the case of contact:

\[ M(q) \ddot{q} = \tau - C(q, \dot{q}) \dot{q} - G(q) - F(\dot{q}) - J_{ee,e}^T(q) \cdot F_e - J_{ee,f}^T(q) \cdot F_f \]

\[ F_e = K_e \cdot \delta^n + \lambda_e \cdot \dot{\delta} \cdot \delta^n \]

\[ F_f = F_n \cdot (\mu \cdot \sigma(\nu_{r,ee}) + c \cdot \nu_{r,ee}) \]

\[ \dot{\tau} = S(q_d, \dot{q}_d, \ddot{q}_d, q, \dot{q}, \ddot{q}, \tau) \]

\( (6 \times 2) + 1 + 2 + 6 \) equations

\( \delta = x_{ee} - x_0, \dot{\delta} = \dot{x}_{ee} \)

\( \nu_{r,ee} = (\dot{y}_{ee}, \dot{z}_{ee}) \)

\[ J_{ee,e}^T = \begin{bmatrix} \frac{\partial x_{ee}}{\partial q_1} & \frac{\partial x_{ee}}{\partial q_2} & \frac{\partial x_{ee}}{\partial q_3} & \frac{\partial x_{ee}}{\partial q_4} & \frac{\partial x_{ee}}{\partial q_5} & \frac{\partial x_{ee}}{\partial q_6} \end{bmatrix} \]

\[ J_{ee,f}^T = \begin{bmatrix} \frac{\partial y_{ee}}{\partial q_1} & \frac{\partial y_{ee}}{\partial q_2} & \frac{\partial y_{ee}}{\partial q_3} & \frac{\partial y_{ee}}{\partial q_4} & \frac{\partial y_{ee}}{\partial q_5} & \frac{\partial y_{ee}}{\partial q_6} \\
\frac{\partial z_{ee}}{\partial q_1} & \frac{\partial z_{ee}}{\partial q_2} & \frac{\partial z_{ee}}{\partial q_3} & \frac{\partial z_{ee}}{\partial q_4} & \frac{\partial z_{ee}}{\partial q_5} & \frac{\partial z_{ee}}{\partial q_6} \end{bmatrix} \]

switching conditions: if \( F_e < 0 \), then contact is broken
Radau5

- Radau5 solver of Hairer (2002)
- expects constant mass matrix
- can handle DAE’s

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q} \\
\ddot{q} \\
\dot{\tau} \\
\ddot{\delta} \\
\dot{\delta} \\
\ddot{\delta}
\end{bmatrix}
= 
\begin{bmatrix}
\dot{q} \\
\ddot{q} \\
M(q) \cdot \ddot{q} - \tau - D(q, \dot{q}) \\
S(q_d, \dot{q}_d, \ddot{q}_d, q, \dot{q}, \ddot{q}, \tau) \\
K_e \cdot \delta^n + \lambda_e \cdot \dot{\delta} \cdot \delta^n
\end{bmatrix}
\]

- if contact is made, last two dimensions are dropped and environment and friction force are taken into account in $D(q)$.
- implicit form for environment model.

Multibody dynamics
Switching

- switching conditions are monitored.
- when a switch is detected, the time interval containing the switch is recalculated with a small time-step and the switching point is determined more accurately.
- there are better ways to handle this.
Explicit or implicit Euler

- alternatively, explicit or implicit Euler may be used.

\[ \ddot{q} = M^{-1}(q) \cdot (\tau + D(q, \dot{q})) \]
\[ \dot{\tau} = S(q_d, \dot{q}_d, \ddot{q}_d, q, \dot{q}, \ddot{q}, \tau) \]

- if contact is made, environment and friction force are taken into account in \( D(q) \).
- this was used to verify the correctness of the Radau5 approach
Considered cases

- case 1: $q_d$ trajectory data from a writing task.
- case 2: $q_d$ trajectory data from a writing task augmented with an excitation signal.
- purpose: to estimate:
  - the environment parameters: stiffness, damping, ... 
  - friction parameters.
  - orientation and position of the environment.
from the measured forces on the end-effector and the (discretised) end-effector position.
Some results

**Force deformation hysteresis loop**

![Graph showing environment deformation versus environment force](image-url)

- **Multibody dynamics**
- **Environment deformation versus environment force**

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**Notes:**
- Force vs. Position graph indicating hysteresis in the system.
Some results

**Joint angles**

- Joint angle for axis 1
  - $q_1$
  - $q_{d,1}$

- Joint angle for axis 2
  - $q_2$
  - $q_{d,2}$

- Joint angle for axis 3
  - $q_3$
  - $q_{d,3}$

- Joint angle for axis 4
  - $q_4$
  - $q_{d,4}$

- Joint angle for axis 5
  - $q_5$
  - $q_{d,5}$

- Joint angle for axis 6
  - $q_6$
  - $q_{d,6}$
Estimated environment parameters

Estimated exponent

- true exponent
- estimated exponent (no persistent excitation)
- 99% uncertainty bounds

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Conclusions

- minimal coordinates approach was the most elegant approach for this case.
- working with constraints (at acceleration level) requires symbolic tools.
- quite complex models can be handled through a systematic approach.
- switching between models involves careful thought: easier with minimal coordinates, quite tedious when using constraints.
- persistent excitation allows for better estimation.
Questions

Thank you for your attention!

Questions?
Joint torques

Multibody dynamics
Force deformation hysteresis loop

Environment deformation - force hysteresis loop

Multibody dynamics
Loss of contact

Environment deformation and force

- end-effector penetration
- environment deformation

Environment force

Multibody dynamics
Estimated environment parameters

Estimated stiffness coefficient

- True stiffness coefficient
- Estimated stiffness coefficient (no persistent excitation)
- 99% uncertainty bounds

Multibody dynamics
Estimated environment parameters

Estimated damping coefficient

true damping coefficient
estimated damping coefficient (no persistent excitation)
99% uncertainty bounds

Multibody dynamics
Estimated environment parameters

**Estimated x-component**

- true x-component
- estimated x-component (no persistent excitation)
- 99% uncertainty bounds

**Position (m)**

0 0.5 1

0 1 2 3 4 5 6 (time (s))

**Multibody dynamics**
Estimated environment parameters

Multibody dynamics
Estimated environment parameters

Estimated z-component

- true z-component
- estimated z-component (no persistent excitation)
- 99% uncertainty bounds

Estimated z-component

- true z-component
- estimated z-component (persistent excitation)
- 99% uncertainty bounds

Multibody dynamics


to Dynamic Trajectory Compensation.