Computer-aided analysis of multibody dynamics (part 2)

Railway vehicle dynamics

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- Part I: Wheel-rail contact in railway dynamics
  - Contact geometry
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  - Wheelset dynamic behavior
- Part II: Railway dynamics - multibody approach
  - Multibody representation
  - Wheel/rail contact – independent wheel model
  - Flange contact model
  - Validations - Applications
  - Other topics
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Contact geometry

Wheelset with conical wheel on « knife-edge » rails

- Rolling radii:
  \[ r_r = r_o + \delta_y y \]
  \[ r_i = r_o - \delta_y y \]
  (In practice: \( \delta_y \approx 1/20 \) ... 1/40)

- Equivalent conicity:
  \[ \lambda = \delta_y \]

- Rolling radii difference (left-right):
  \[ \frac{\Delta r}{2} = \lambda y \]

- Roll angle:
  \[ \varphi = \frac{\Delta y}{2l_o} = \frac{\lambda y}{l_o} \]
Contact geometry

Wheelset kinematic motion (~ first order kinematics)

- Hypothesis: pure rolling motion (no slip)
  - Kinematic relations (first order)
    \[
    \begin{align*}
      V_y &= \omega r_y \\
      V_{\psi} &= \omega r_{\psi}
    \end{align*}
    \]
  - Kinematic relations (first order)
    \[
    V = \frac{V_y + V_{\psi}}{2} = \omega r_0
    \]
- Constraints (in \( y \) and \( \psi \)):
  - Lateral (\( y \)):
    \[
    \dot{y} = V \sin \psi = V \psi
    \]
  - Yaw (\( \psi \)):
    \[
    \dot{\psi} = \frac{V_{\psi} - V_y}{2I_0} = \frac{\omega \Delta \rho}{I_0 r_0} = \frac{V}{2I_0 r_0} y
    \]

Contact geometry

Wheelset kinematic motion (next)

- 2 d.o.f. kinematic equations (= constraints at « velocity level »)
  \[
  \begin{pmatrix}
    \dot{y} \\
    \dot{\psi}
  \end{pmatrix} =
  \begin{pmatrix}
    0 & V \\
    V & 0
  \end{pmatrix}
  \begin{pmatrix}
    y \\
    \psi
  \end{pmatrix}
  \]
- Periodic Solution:
  \[
  \begin{align*}
    y &= A \sin(2\pi f t) \\
    \psi &= A \frac{\lambda}{I_0 r_0} \cos(2\pi f t)
  \end{align*}
  \]
  - Frequency:
    \[
    f = \frac{V}{2\pi \sqrt{\frac{\lambda}{I_0 r_0}}}
    \]
  - Wave length:
    \[
    L_{\text{wave}} = \frac{V}{f} = 2\pi \sqrt{\frac{I_0 r_0}{\lambda}}
    \]
Contact geometry

Bogie: yaw kinematic motion (similar reasoning)

- Wave length:
  \[ L_{\text{wave}} = L_{\text{roll}} \sqrt{1 + \left( \frac{l}{l_0} \right)^2} \]
  \( l \): half wheelset base
  \( l_0 \): half wheelset length

Wheel/rail with circular profiles (as a matter of interest)

- More complex kinematics:
  - Roll angle:
    \[ \varphi = \frac{y \delta_a}{l_0 - r_0 \delta_a} \]
  - Vertical disp.:
    \[ z = \frac{y^2}{2(R^- - R^+)} \left( \frac{l_0 + R^+ \delta_a}{l_0 - r_0 \delta_a} \right)^2 \]
  - Equiv. conicity:
    \[ \lambda = \frac{R^- \delta_a}{R^- - R^+} \left( \frac{l_0 + R^+ \delta_a}{l_0 - r_0 \delta_a} \right) \]

Contact geometry

Wheel/rail contact: yaw angle influence on contact geometry

- Longitudinal shift of the contact point (\( \theta \) « shift angle») due to wheelset yaw + wheel conicity

- Insignificant for the tread contact point (angle ~ 0.05 rad)
- Fundamental for the flange contact point (angle > 1 rad): see latter on
Contact geometry

Real wheel profiles: numerical solution

- Profile definition (analytical form or numerical measurements)
  - ex. UIC 60 rail (piecewise circular), S1002 wheel (piecewise polynomials)
- Numerical determination of the contact(s) point(s) = f(y, ψ)
  - Pre-computation (look-up tables => f(y, ψ))
  - In-line computation (Newton-Raphson, dichotomic method, ... see Part II)
- Computation of the left/right rolling radii r and contact angles δ:

![Graphs showing Δr and Δδ vs. y](image)

Contact geometry

Double (flange) contact point

- Tramway « clear » double contact (tread + flange)
- Train the contact point « moves » from tread to flange (with possible jump !!)

![Diagram showing contact points](image)
Contact geometry

Double (flange) contact point

- Tramway ⇔ « clear » double contact (tread + flange)
- Train ⇔ the contact point « moves » from tread to flange (with possible jump !!)

Contact point « jump »

- Exists on theoretical « new » profiles (ex. S1002 wheelset on UIC60 rail)
  - New rail :
  - Slightly worn rail :

- Exists also on worn profile (but their location change with wear !)
Contact forces

Coulomb's law:

- No slip \( \Rightarrow \vec{F}_f = \mu |\vec{F}_n| \)
- Slip \( \Rightarrow \vec{F}_f < \mu |\vec{F}_n| \)

Creepage (« between slip and pure rolling »)

- Longitudinal creepage: \( \gamma_x = \frac{V_i^r - V_i^s}{V} \)
- Lateral creepage: \( \gamma_y = \frac{V_i^l - V_i^s}{V} \)
- Spin creepage: \( \gamma_{\psi} = \frac{\Omega_i^r - \Omega_i^s}{V} \)

\( V = \frac{V_i^r + V_i^s}{2} \) \hspace{1cm} \text{Point of contact assumption !!!!!}
Contact forces

Creepage computation (not MBS approach!)

- Longitudinal creepage:
  \[ \gamma_x = \frac{V - l_0 \dot{\psi} + r_p \dot{\theta}}{V} \]

- Lateral creepage:
  \[ \gamma_y = \frac{(\dot{y} - V \sin \psi - r_p \dot{\phi}) \cos(\delta_r - \varphi)}{V} \]

- Spin creepage:
  \[ \gamma = \frac{\dot{\psi} \cos(\delta_r - \varphi) + \dot{\theta} \sin \delta_r}{V} \]

Contact forces

Contact surface

- Non uniform pressure distribution

- Contact ellipse (a, b) \(\bowtie\) Hertz (but + creepage)
  \[ a = m \left( \frac{3(1 - \sigma^2)}{2E(A + B)} \right)^{1/2} F_n^{1/3} \]
  \[ b = n \left( \frac{3(1 - \sigma^2)}{2E(A + B)} \right)^{1/3} F_n^{1/3} \]
  A, B, m, n depend on contact curvature radii
Contact forces

Kalker’s theory

- Linear theory
  \[
  \begin{pmatrix}
  F_x \\
  F_y \\
  M_z
  \end{pmatrix} =
  \begin{pmatrix}
  -k_{11} & 0 & 0 \\
  0 & -k_{22} & -k_{23} \\
  0 & k_{32} & -k_{33}
  \end{pmatrix}
  \begin{pmatrix}
  \gamma_x \\
  \gamma_y \\
  \gamma_z
  \end{pmatrix}
  \]

  with:
  \[ k_{11} = \alpha h G_{11} \]
  \[ k_{22} = \alpha h^3 G_{22} \]
  \[ k_{33} = \alpha h^5 G_{33} \]
  \[ k_{32} = \alpha h^3 G_{32} \]

- Heuristic modeling of saturation

  \[ F' = \sqrt{(F_x')^2 + (F_y')^2} \]

  - if \( F' \leq \mu F_a \) \( F' = \mu F_a \)
  
  - if \( F' > \mu F_a \) \( F' = \frac{F'}{\mu} \frac{1}{2 \mu} \left( F' \right)^2 + \frac{1}{2 \mu} \left( F' \right)^2 \)

- Non linear theories: Duvorol, Fastsim (Kalker), Contact (Kalker)

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Wheelset dynamic behavior

Linear model

- Hypotheses:
  - « suspended » wheelset
  - Linearized equations of motion:

\[
\begin{pmatrix}
  m & 0 \\
  0 & 1
\end{pmatrix}
\begin{pmatrix}
  \frac{d^2 y}{dt^2} \\
  \frac{d \theta}{dt}
\end{pmatrix} + \begin{pmatrix}
  0 & -k_y \\
  -k_{\theta} & \frac{1}{J}
\end{pmatrix}
\begin{pmatrix}
  \frac{d y}{dt} \\
  \frac{d \theta}{dt}
\end{pmatrix} + \begin{pmatrix}
  0 & 1 \\
  1 & 0
\end{pmatrix}
\begin{pmatrix}
  V_c + 2k_{\theta} \frac{d \theta}{dt} \frac{d \theta}{dt} + k_{\theta} \theta \\
  V_c + 2k_y \frac{d y}{dt} + k_y \theta
\end{pmatrix}
\begin{pmatrix}
  y \\
  \theta
\end{pmatrix} = 0
\]

- Solution: \( y = \sum \gamma_f e^{\lambda_f t} \) et \( \theta = \sum \gamma_{\theta f} e^{\lambda_{\theta f} t} \)

=> « Linear » Critical speed \( V_{\text{Lim}} \):

- « Hunting » eigenmode
  - Eigenvalue \( \lambda \) versus speed:

Wheelset dynamic behavior

Linear model: example

- ROBOTRAN/MBsysLab model:

=> Comparison between linear – non linear model
Wheelset dynamic behavior

Linear model: example

- ROBOTRAN/MBsysLab model:
  http://www.prm.ucl.ac.be/FDP/Tutorial/Medium/Bogie/Introduction.html

=> Behavior at low and high velocity

![Graphs showing hunting mode evolution versus velocity](image)

Wheelset dynamic behavior

Linear model: example

- ROBOTRAN/MBsysLab model:
  http://www.prm.ucl.ac.be/FDP/Tutorial/Medium/Bogie/Introduction.html

=> Evolution of the hunting mode versus velocity (via successive modal analyses)

![Graph showing modal eigenvalue evolution](image)
Wheelset dynamic behavior

Non-linear model

- Notion of limit cycle (ex. van der Pol differential equation)

\[ \ddot{y} - \xi \omega_0 (1 - y^2) \dot{y} + \omega_0^2 y = 0 \]

\[ \omega_0, \xi > 0 \]

- small \( y \) => negative damping \( \Downarrow \) the system stores energy
- large \( y \) => positive damping \( \Downarrow \) the system loses energy

- Limit cycle: oscillatory solution in which the 2 energies compensate

- Application: limit cycle of a « suspended » wheelset:

Contact geometry

Critical speed \( V_{cr} \)

- Limit cycle amplitudes versus system velocity (system = wheelset, amplitude = \( y \))

- Numerical determination of the critical speed \( V_{cr} \) and \( V_{lim} \)
  - Time simulation from high speed (with lateral « safety bumps ») to low speed
Conclusions (part I)

Key points

- Contact geometry: a difficult problem to solve
- Contact forces: idem • thanks ! Mr Kalker (et al.)
- Dynamic behavior of a wheeset on a straight track: critical speeds

Other aspects

- Curving dynamics: « crab-wise » bogie behavior
- Derailment criteria (Y/Q wheel force ratio)
- Wear
- Track irregularities => noise, comfort, ...
- Full train: different morphologies, tilting trains, pneumatic suspensions, ...

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The « BAS2000 » bogie: a general case

Six «3D» kinematic loops

=> No more wheelset, even artificial (left-right)

MBS: wheel or wheeset - does’nt matter!

In relative coordinates, a wheel is a leaf body to which forces apply

Independent wheels

Wheelset = (axle + 2 « locked » indep. wheels)
The « BAS2000 » bogie: a general case

- A wheel = a leaf body which is constrained on a rail (inertial body)

- The BAS2000 cannot be modeled as a bogie … but as a mechanism with wheels! => relative coordinates approach suitable

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The « wheel/rail » joint »

=> 5 dof + 1 « normal » constraint

Recall : coordinate partitioning reduction

Coordinate partitioning:

\[
\begin{align*}
(M_{uu} + M_{uv} B_{vn} + B_{vu}^T M_{uvu} + B_{vv}^T M_{vvn} B_{vn}) \ddot{u} \\
- (M_{uu} + B_{vu}^T M_{vuu} (J_v)^{-1} b + c_v + B_{vu}^T c_v = Q_u + B_{vu}^T Q_v
\end{align*}
\]

\[
\Leftrightarrow \quad M_{red}(u,v,t) \ddot{u} + c_{red}(\dot{u}, u, v, t) = Q_{red}
\]

ODE!
A Single Wheel on a Rail : 5 d.o.f.

- 2 Auxiliary variables:
  - \( w \) : lateral position
  - \( \theta \) : Shift angle

3-D Contact Constraints:

1. Point Q belongs to Rail Surface
   \[ h_1(q, w, \theta) \triangleq u_3 - \mu(u_2) = 0 \]

2. Wheel / Rail Surfaces Tangent at Point Q
   \[ h_2(q, w, \theta) \triangleq \hat{T}_3 \cdot \hat{R}_1 = 0 \]
   \[ h_3(q, w, \theta) \triangleq \hat{T}_3 \cdot \hat{R}_2 = 0 \]
   \[ \Rightarrow \text{Lagrange Multipliers Technique} \]

Wheel on rail: Constraints solution

Robust solution of contact constraints
\[ h_1(q, w, \theta) \triangleq u_3 - \mu(u_2) = 0 \]
\[ h_2(q, w, \theta) \triangleq \hat{T}_3 \cdot \hat{R}_1 = 0 \]
\[ h_3(q, w, \theta) \triangleq \hat{T}_3 \cdot \hat{R}_2 = 0 \]

A dichotomic method "embedded into" the Newton-Raphson algorithm
Wheel on rail: Constraints solution

\[ \begin{align*}
\frac{dh}{dv^T} &\sim \frac{\partial h}{\partial v^T} + \left( \frac{\partial h}{\partial w} \right) \frac{\partial w}{\partial v^T} + \left( \frac{\partial h}{\partial \theta} \right) \frac{\partial \theta}{\partial v^T} \\
\text{requires} &\quad = 0 \text{ when the constraints are satisfied}
\end{align*} \]

Wheel on rail: Constraints derivative

\[ \dot{h}_1 = \frac{d}{dt} \left( u \cdot \dot{\lambda}_3 \right) - \frac{d}{dt} \left( \mu (u \cdot \dot{\lambda}_2) \right) = \dot{u} \cdot \dot{\lambda}_3 - \mu' (u_2) \dot{u} \cdot \dot{\lambda}_2 = 0 \]

\[ \dot{h}_1 = \frac{1}{\cos \alpha} \left( \dot{u} \cdot \dot{\lambda}_3 \right) = 0 \]

\[ \cos \alpha \neq \frac{\pi}{2} \quad \dot{u} \cdot \dot{\lambda}_3 = 0 \quad \lambda_1 = F_N \]

\[ \left( \dot{x} + \frac{u_2}{\cos \beta} T_2 + \omega^X \times w + \dot{\theta} \rho(\theta) T_1 \right) \cdot \dot{R}_3 = 0 \]

\[ h(q) \text{ satisfied} \quad (\dot{x} + \omega^X \times w) \cdot \dot{R}_3 = 0 \]

\[ J(q, w(q), \theta(q)) \dot{q} = 0 \]
Wheel on rail: Constraints 2\textsuperscript{d} derivative

\[ \ddot{u} \cdot \dot{R}_3 = 0 \]

\[ \frac{d}{dt} (u \cdot \dot{R}_3) = u \cdot \dot{R}_3 + u \cdot (\omega^R \times \dot{R}_3) \]

\[ (x + (\dot{\omega}^X + \ddot{\omega}^X \cdot w + \omega^X \cdot \left( \theta \rho(w) \mathbf{T}_1 + \frac{\omega}{\cos \beta} \mathbf{T}_2 \right)) \cdot \dot{R}_3 - (\omega^R \cdot (x + \omega^X \cdot w)) \cdot \dot{R}_3 = 0 \quad (19) \]

\[ J \ddot{q} + \left( \frac{\partial J}{\partial \dot{q}} \ddot{q} + \frac{\partial J}{\partial \dot{w}} \ddot{w} + \frac{\partial J}{\partial \theta} \right) = 0 \]

Wheel on curved rail
Wheel on curved rail

1. point \( Q \) belongs to the (straight) rail surface:

\[
h_1^Q(q, w, \theta) = u^* \cdot \dot{I}_2^* - \mu \left( u^* \cdot \dot{I}_2^* \right) = 0
\]

where \( \mu \) is the describing function of the local straight track rail profile, defined in the \( \{F^1\} \) frame.

2. the surfaces of the wheel and rail must be tangent at the contact point:

\[
h_2^Q(q, w, \theta) = h_2 \quad \text{with } h_2 \text{ given by } 5 \quad (25)
\]

\[
h_3^Q(q, w, \theta) = h_3 \quad \text{with } h_3 \text{ given by } 6 \quad (26)
\]

where \( w, \theta \) are the auxiliary variables already defined for the straight track.

Creepages : a « real » MBS computation

Longitudinal creepage:

\[
\xi_x \triangleq \frac{(\dot{x} + \omega^Y \times w) \cdot \dot{T}_1}{\dot{x} \cdot \dot{R}_1}
\]

Lateral creepage:

\[
\xi_y \triangleq \frac{(\dot{x} + \omega^Y \times w) \cdot \dot{T}_2}{\dot{x} \cdot \dot{R}_1}
\]

Spin creepage:

\[
\xi_{\omega^Y} \triangleq \frac{\omega^Y \cdot \dot{T}_3}{\dot{x} \cdot \dot{R}_1}
\]
Wheel on rail: Kalker’s model

\[ F_w = F_{long} T_1 + F_{lat} T_2 + F_{vert} T_3 \quad \text{and} \quad M_w = M_{spin} T_0 \]

Kalker (linear):

\[
\begin{pmatrix}
F_{long} \\
F_{lat} \\
M_{spin}
\end{pmatrix} =
\begin{pmatrix}
-(a b) C_{sh} c_{11} & 0 & 0 \\
0 & -(a b) C_{sh} c_{22} & -(a b)^3 / 2 C_{sh} c_{33} \\
0 & (a b)^3 / 2 C_{sh} c_{22} & -(a b)^2 C_{sh} c_{33}
\end{pmatrix}
\begin{pmatrix}
\xi_u \\
\xi_w \\
\xi_{tw}
\end{pmatrix}
\]

Wheel on rail: flange forces (tramways)
Wheel on rail: flange forces

Influence of the shift angle

For the flange contact force, in terms of tangent slip, the tread contact («creepage») can be seen as a pure rolling motion (~ICR)

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Geometrical validation

Wheelset limit cycle

Limit cycle of a rigid wheelset at constant speed
Bogie: « non linear » critical speed

The “BAS 2000” articulated bogie

GraSMech – Multibody – Part II

The ROBOTRAN results (AVSS benchmark G2, 1991)
"BAS 2000" : straight track behavior

« Ping-pong » effect on a straight track

Yaw deformation

Clearance = 9 mm
Clearance = 5 mm
Clearance = 2 mm

0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
[sec]

-0.008 -0.006 -0.004 -0.002 0 0.002 0.004 0.006 0.008

The "Tram 2000" tramway

The "Tram 2000"

Model:

Complete TRAM 2000 in Entry Curving:
- 4 Sub-Systems
- 74 Variables
- 32 Constraints
- 42 Degrees of Freedom

Carbody A
Front BAS 2000 Bogie
Carbody C
Carbody B

Conventional Bogie
Front BAS 2000 Bogie

Velocity: 25 km/h
Initial Conditions: Tramway Centered on the Straight Track
“Tram 2000” tramway - results

Result:
Wheel / Rail Alignment in a Low Radius Curve (R = 50 m)

Wheel / Rail Angle of Attack

Comparison of bogie behaviors

Bogies:
- BW: Rigid bogie with wheelets
- BI: Rigid bogie with independent wheels
- BA: BA2000 articulated « bogie »

Situation: entry curving
Comparison of bogie behaviors

Steady state equilibrium: tread and flange lateral forces

- « Crab-wise » behavior
  - BW

- « Crab-wise » behavior
  - BI

- Distorted, no crab-wise
- Dissipated power minimized

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Multibody and railway “electro” dynamics

Railway bogies actuated by three phase induction motors

Problem:
Torque oscillations lead to fatigue stress in the chassis

- $F_z$ in the motor to chassis-joint
- Electromechanical torque $C_m$

Multibody and railway pneumatic suspension

Multibody system

Pneumatic system

Hybrid system

=> Next lecture