Computer-aided analysis of rigid and flexible multibody systems

Organization and Teaching team

- Part I (2005-2006): Modelling approaches and numerical aspects
  - Speakers:
    - Dr. Olivier Brüls, OB, ULg
    - Dr. Gaëtan Kerschen, GK, ULg
    - Prof. Paul Fisette, PF, UCL
    - Dr. Joris Peeters (replacing Prof. Wim Desmet), JP, KUL
    - Prof. Jean-Claude Samin, JCS, UCL
    - Prof. Olivier Verlinden, OV, FPMs
- Part II (2006-2007): Flexible systems and special topics

Schedule

Lessons on wednesday from 14.00 to 18.00

- Lesson 1 (February 15):
  - Part one: Introduction (JCS) + applications (PF,OB,GK,OV)
  - Part two: approach based on Minimal coordinates (OV)
- Lesson 2 (February 22): approach based on Cartesian coordinates (OV,JP)
- Lesson 3 (March 1): approach based on Finite elements (OB,GK)
- Lesson 4 (March 8): approach based on Relative coordinates (PF,JCS)
- Lesson 5 (March 15): numerical problems (integration,...)
- After Easter (April 26?): presentation of projects by students
Introduction

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Introduction: multibody applications

- Mechanisms
- Railway vehicles
- Space applications
- Set of articulated bodies
- Robot manipulators

What is a multibody system?

Multibody System (MBS):

- Bodies (rigid or flexible)
- Joints
- Force elements
Principle

System defined from technical elements

Equations of motion built (and eventually solved) automatically

Purpose of simulation

- To assess the behaviour of a system before its construction
  - so as to dimension its mechanical elements
  - so as to optimize its performances
  - so as to design a controller
  - ...
- To identify possible problems on an existing system

MBS versus FEM

Why do we need to develop a new theory whereas structural dynamics and finite element theories are well established?

Finite elements
- Flexible bodies
- Structural and modal analysis (small motions)
  \[ Mq + Cq + Kq = p(t) \]

Multibody approach
- Rigid bodies
- Large motions
  \[ M(q)\dot{q} + c(q,\dot{q},t) = Q + J^T\lambda \]
  \[ g(q,t) = 0 \]

MBS with flexible bodies
limit: efficiency versus universality
Historical origins

The American *Explorer I* flew in January 1958. *Explorer I* was long and narrow like a pencil. It was supposed to rotate around its own centerline, like a pencil spinning about its lead. It was definitely not supposed to rotate end over end, like an airplane propeller or a windmill blade.

Some unstable satellites
Historical origins

First publications:


Different approaches of MBS formalisms

- Choice of coordinates
- Mechanical principle to generate the equations
- Computer implementation

Choice of coordinates

Absolute coordinates
(a bad example...)

- 6 bodies:
  => 36 differential eq. of motion
- 6 revolute joints:
  => 30 algebraic equations

TOTAL: 66 equations (DAE) for a 6 d.o.f. system
Choice of coordinates

**Absolute coordinates**
(a good example…)

- Main body: 6 absolute coordinates
- Each bogie frame: 6 absolute coordinates
- Each wheelset: 6 absolute coordinates
- All bodies "elastically" inter-connected

6n equations (DAE) for a 6n d.o.f. system

---

Choice of coordinates

**Relative coordinates**
(a good example…)

- 6 d.o.f.
- 6 relative coordinates

=> 6 ODE

---

Choice of coordinates

Absolute or Relative coordinates?
(a medium example... = reality)

1 d.o.f.

- either:
  - 3 relative coordinates
  - 2 constraints
  => 5 DAE

- or:
  - 5 absolute coordinates

4 constraints

=> 9 DAE

or:

- 1 "minimal" coordinate
- 0 constraints

=> 1 ODE
Choice of coordinates

Data input:
of the reference configuration

Absolute:
• for each center of mass: \(x, y, z\)
• for each body frame: \(\theta_1, \theta_2, \theta_3\)

Relative:
• \(x, y, z\), but in a local body frame
• relative orientation of frames

Conclusions

Influence on:
• Number of equations \(\leftrightarrow\) computer efficiency
• +/- user-friendly
• Kinematical and dynamic equations +/- complicated
• +/- easy for MBS computer implementation
• = leads to DAE equations

An alternative approach:
"Natural" coordinates

Rigid body \rightarrow Set of equivalent punctual masses

No need of rotational coordinates!

Used by:
- Garcia de Jalon
- Nikravesh
Different approaches of MBS formalisms

- Choice of coordinates
- Mechanical principle to generate the equations
- Computer implementation

Which formalism?

- Newton – Euler (basic)
- Virtual work or power principle
- Lagrange equations
- Recursive Newton-Euler algorithm
- Recursive “order N” algorithm

Which formalism?

Description of a multibody system (basic elements)

- Topology

Example: “tree-like”

Reference attachment points

Bodies

Joints

Base (initial body)
Which formalism?

Description of a multibody system (basic elements)

- **Bodies**, defined by:
  - mass
  - position of the center of mass
  - body-fixed frame
  - inertia tensor (with respect to the body-fixed frame)
  - auxiliary body-fixed frames (attachment points)

- **Ground** = fixed reference body (except for space applications)

Which formalism?

Description of a multibody system (basic elements)

- **Joints**, defined between frames attached to bodies
  - revolute
  - prismatic

  ![Revolute Joint](attachment://revolute.png)

  ![Prismatic Joint](attachment://prismatic.png)

  1 relative d.o.f.
  - (rot)
  - (trans)

Which formalism?

Description of a multibody system (basic elements)

- **Joints**, defined between frames attached to bodies
  - universal
  - spherical
  - cylindrical

  ![Universal Joint](attachment://universal.png)

  ![Spherical Joint](attachment://spherical.png)

  ![Cylindrical Joint](attachment://cylindrical.png)

  2 relative d.o.f.
  - (2 rot)
  - (2 rot + 1 trans)

  3 relative d.o.f.
  - (3 rot)

  2 relative d.o.f.
  - (1 rot + 1 trans)
Which formalism?

Description of a multibody system (basic elements)

- Joints, defined between frames attached to bodies
  - Planar
  - Helicoidal (screw)
  - Contact joint

  
  3 relative d.o.f. (1 rot + 2 trans)
  1 relative d.o.f. (1 rot/ trans)
  5 relative d.o.f. or 3 relative d.o.f. (rolling without slip)

Which formalism?

Description of a multibody system (basic elements)

- Force elements, defined between bodies

  Example: spring and damper suspension elements

Which formalism?

- Newton – Euler (basic)

  ▶️ For each individual body, write:

  Newton (vector) equation: \( \sum F = m \ddot{x} \)

  Euler (vector) equation: \( \sum L = \dot{H} \)

  … easy if absolute coordinates are used …

  ▶️ Collect all equations
Which formalism?

- Newton – Euler (basic)
  - Collect all equations
    \[
    \begin{bmatrix}
    \ddot{\mathbf{x}} \\
    \ddot{\mathbf{\dot{x}}}
    \end{bmatrix}
    = \mathbf{F}
    \]
    rely on sparse-oriented numerical methods

  - Collect all equations in your computer program
    \[
    \mathbf{M} \ddot{\mathbf{q}} + \cdots = \mathbf{F}_{\text{con}} + \mathbf{F}_{\text{cons}}
    \]
    1 contributive torque component
    (spring, friction, actuator, …)
    5 constraint force/torque components

- Newton – Euler (basic)
  - Eliminate (numerically) all constraint force/torque component
    \[
    \mathbf{J} \mathbf{M} \ddot{\mathbf{q}} + \cdots = \mathbf{J} \mathbf{F}_{\text{con}}
    \]
  - Complete the dynamical set of equations in your computer program with the kinematical joint constraint equations
    \[
    \text{... lead to a huge amount of equations but relatively simple to obtain ...}
    \]
Which formalism?

- Newton – Euler (modified)

  - for each joint, sum analytically
    - all equations of the « descendant » bodies
  - all internal force/torque of the appending subsystem disappear

Translational (vector) joint equation:

\[ F^i = f^i(q, \dot{q}, \ddot{q}, F_{ext}) \]

Rotational (vector) joint equation:

\[ L^i = h^i(q, \dot{q}, \ddot{q}, F_{ext}, L_{ext}) \]

- project analytically these vector equations onto the joint axis
- the joint constraint force/torque components also disappear

Implement these analytical results in your computer program

- either in implicit form (inverse dynamical model)
  \[ Q = \Phi(q, \dot{q}, \ddot{q}, F_{ext}, L_{ext}) \]
- or in semi-explicit form (direct dynamical model)
  \[ \dot{Q} = M(q) \dot{q} + C(q, \dot{q}, F_{ext}, L_{ext}) \]
Which formalism?

- Virtual power (or work) principle

close to the modified Newton-Euler approach
(and historically inspired by it)

Translational vector equation

虚位移原理

Rotational vector equation

虚位移原理

Rotational vector equation

虚位移原理
Which formalism?

- Virtual power (or work) principle
  - project analytically these vector equations onto the joint axis
  - expand the r.h.s. of the equations in terms of the generalized coordinates and their derivatives
  - implement these analytical results in your computer program

$$Q = M(q) \ddot{q} + c(q, \dot{q}, F_{\text{ext}}, \mathbf{r}_{\text{ext}}, g)$$

Which formalism?

- Recursive Newton-Euler algorithm
  - especially convenient for relative coordinates
  - first develop a « forward » kinematical recursion

$$A^o = A^o \cdot A^i(q^i)$$

$$\dot{q}^j = \dot{q}^j + \Omega(q^j) + \cdots$$

$$\dot{\omega}^j = \dot{\omega}^j + \cdots$$

$$\ddot{\omega}^j = \ddot{\omega}^j + \cdots$$

Which formalism?

- Recursive Newton-Euler algorithm
  - especially convenient for relative coordinates
  - first develop a « forward » kinematical recursion

$$p^j = p^j + z^i(q^i) + \cdots$$

$$\dot{p}^j = \dot{p}^j + \cdots$$

$$\ddot{p}^j = \ddot{p}^j + \cdots$$
Recursive Newton-Euler algorithm

- second, develop a « backward » dynamical recursion in vector form

\[ F_i = F_i^0 + m_i \ddot{x}_i \ldots \]
\[ L_i = L_i^0 + I_i \dot{\omega}_i \ldots \]

Recursive Newton-Euler algorithm

- project analytically these vector equations onto the joint axis
  - l.h.s. \( F_i \cdot \dot{q} + L_i \cdot \ddot{\omega}_i = \dot{Q}_i \)
  - r.h.s. \[ Q_i = \Phi (\dot{x}_i, \dot{\omega}_i, \ldots, F_i, L_i) \]

Which formalism?

- finally, collect all these dynamical equations:
  \[ \dot{Q} = \Phi(q, \dot{q}, \ddot{q}, F_{ext}, L_{ext}, \ddot{\omega}) \]

implicit form (inverse dynamical model)

- note : the semi-explicit form can also be obtained by isolating \( \ddot{q} \)

- a first « forward » kinematical recursion step

- a second « backward » dynamical recursion step

- a third « forward » recursion step

\[ \ddot{q} = M^{-1} (q) \left[ Q - c(q, \dot{q}, F_{ext}, L_{ext}) \right] \]

explicit form of the dynamical model
Which formalism?

Comparison of computational costs
(for simulation purpose: explicit form)

Example: a binary tree multibody system

Different approaches of MBS formalisms

- Choice of coordinates
- Mechanical principle to generate the equations
- Computer implementation
**Computer implementation**

- **Numerical approach**
  - MBS description
    - set of numerical data
    - including:
      - the particular topology
      - constant values
      - initial values $q, \dot{q}$
  - generate numerically all system equations according to the generic MBD formalism
  - Initial conditions
  - solve the system numerically
  - Iterate

**Numerical implementation**

**Different approaches of MBS formalisms**

- **Symbolic approach**
  - Generate all system equations:
    
    $M(q)$ and $c(\dot{q}, q)$
    
    $h(q)$ and $J(q)$
  - can also be done by implementing the MBD formalism in a symbolic computer program

**Symbolic implementation**

**Numerical implementation**

**Different approaches of MBS formalisms**

- **Symbolic approach**
  - MBS description
    - set of data
    - including:
      - the particular topology
      - symbols and zeros
  - generate symbolically all system equations according to the generic MBD formalism
  - Specific model equations, given as a sub-routine
  - Solve iteratively
Principal simulation software

- MSC / ADAMS (1973, USA, reference in road vehicles)
- LMS / DADS (Belgium, originally USA)
- SIMPACK (DLR, Germany, leader in railway dynamics)

In Belgium:
- SAMCEF / MECANO (Samtech/ULg, based on finite elements)
- ROBOTRAN (UCL, advanced symbolic generation of equations of motion)
- EasyDyn (FPMs, free library (GPL) developed for teaching)

Application examples

GraSMech course 2005-2006

Computer-aided analysis of rigid and flexible multibody systems

Applications
Flexible multibody dynamics

- Finite Element method [Géradin & Cardona '01]
- Samcef-MECANO software

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Deployable structures

- Large space structures
- Compact stowed configuration
- Elastic effects:
  - lightweight design
  - pre-stressing
  - small damping

Deployable structures

- Large antenna
- No axial symmetry
- 1 kinematic dof!
- Slow deployment (15 min)
- 1-g test rig ⇒ 0-g behaviour?

Deployable structures

- FE mechanical model
  - Flexibility (struts + panels)
  - Local stiffnesses & friction
  - Inertial forces = negligible
  - 1400 equations
- Kinematic analysis
  - Imposed angle of the central body
  - Driving torque?
  - Forces in the struts?
  - Hinge angles?
Deployable structures

a) 1-g model ⇒ experimental validation
b) 0-g model ⇒ accurate prediction

Deployable structures

Landing gears

- Different phases
  - Deployment / retraction
  - Ground impact
  - Rolling
  - Breaking
  - Taxiing

- Multidisciplinary approach
  - Deformable mechanism
  - Tyre
  - Hydraulics (shock-absorber, brake, steering)
  - Active / semi-active control

Concorde – nose undercarriage
Landing gears

Motivations for modelling:

- Optimal design
  - configuration
  - mass
- Pre-prototyping checks
  - motion
  - loads, stresses
  - stability (shimmy)
  - complementarity with experimental tests

Landing gears

Deployment

Wheel/ground impact

Landing of the A380
Applications

- Hybrid modeling
- Dynamical identification

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Bogie actuation: hybrid modeling

- « Multibody/Electrical » coupled model
  - Symbolic generation of the electro-multibody model
  - Dynamical model of the electrical actuators
  - Equational-level coupling: same « frequency range »

- Application (collab. Bombardier-Transportation, Crespin, France)
  - Articulated bogie (High speed train)
  - 3-phase induction motor
  - Effect of the electrical frequency on fatigue problems (chassis)

Problem: Torque oscillations lead to fatigue stress in the bogie frame during acceleration

« Electro-multibody » model  
Force in the motor to chassis - joint
Semi-active suspension modeling

- « Multibody/Hydraulic/Electrical » coupled model
  - Symbolic generation of the multibody model
  - Dynamical model of the hydraulic system
  - Fast simulation (« real time » in Simulink)

- Application (collab. Tenneco-Automotive, Saint-Trond)
  - Audi A6 (more than 80 relative coordinates)
  - « Kinetic » system: cylinders, pipes, accumulators, …
  - Behavior and sensitivity analysis

Problem: Behaviour prediction of this new suspension system on modern car

Carbody roll
Wrap excitation

Dynamical identification of Tree-like MBS

- « Multibody symbolic dynamical model
  - « Inverse dynamics » model
  - « Reaction dynamics » model
  - Parameter combination: morphology-dependent

- Application (collab. KULeuven)
  - KUKA Robot identification
  - (Human body identification)
**Dynamical identification of Tree-like MBS**

*Serial manipulator*

- ... 6 dof
- « in extenso »
- dynamic model

*Human Body*

- 20 - 30 dof
- « recursive »
- dynamic model

Find ? : Minimal set of dynamical parameters : \( \{p\} \)

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**Dynamical identification of Tree-like MBS**

*Barycentric parameters*

- Barycentric mass : \( \bar{m}_i = \sum_{j \leq i} m_j \)
- Barycentric vector : \( \bar{m}_i \bar{b}_i = \sum_{j \leq i} m_j d_{ij} \)
- Barycentric inertia tensor : \( \bar{K}_i = \bar{I}_i - \sum_{j \leq i} m_j \tilde{d}_{ij} \cdot \tilde{d}_{ij} \)

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**Dynamical identification of Tree-like MBS**

*Recursive formulation (origin : Renaud-88 for linear chains)*

=> Extension to tree-like systems (recursive form)

- \( \varphi(q, \dot{q}, \ddot{q}) \delta = Q \)
Dynamical identification of Tree-like MBS

1. Symbolic Computation of the theoretical combination rules
2. Symbolic Inverse or Reaction Dynamics
3. Symbolic Differentiation w.r. to \( p \)
4. Detection of the « zero » columns in \( D \)
5. Results:

\[ \Rightarrow \text{Minimal Parameters set } \delta^* \]
\[ \Rightarrow \text{Identification Matrix } \phi^* \]

Set of parameters : \( \{p\} \)

<table>
<thead>
<tr>
<th>Minimal set of body ( j ) parameters</th>
<th>Redefinition of parent body ( i ) parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b'_j, b''_j, b_j )</td>
<td>( b'_i := b'_i )</td>
</tr>
<tr>
<td>( K_j, K'_j, K''_j, K'''_j, K'''_j )</td>
<td>( K_i := K_i - b'_j (\phi^* + \phi^* \dot{\phi}^* - K_i \phi^* \dot{\phi}^*) )</td>
</tr>
</tbody>
</table>

with \( K_j' := K_j - K_j'' \)

\( K_i := K_i' + K_i'' \)

Prismatic joint : combination and report rules

Revolute joint : combination and report rules

KUKA IR 361: Dynamic Parameter Estimation

Combining Internal and External models
2D volley-ball motion

Parameter estimation: mean error (%)

Reaction measurement at the foot

Reaction measurement at the trunk

Applications
Railway vehicles

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Building railway vehicle models
**Principal dedicated softwares**

- GENSYS (Sweden)
- MEDYNA (Germany, no longer distributed)
- NUCARS (USA, Association of American Railroads)
- VOCODYM (France, SNCF and INRETS)
- VAMPIRE (UK, AEA Technology)
- SIMPACK (Germany, DLR)
- MSC ADAMS/Rail (USA, MEDYNA routines for contact)

Can be considered as reliable
- ERRI benchmark, Manchester benchmarks
- Comparison with measurements

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**Wheel-rail Contact - Geometric problem**

Contact configuration (position, curvatures, ...) in terms of the lateral displacement

![Diagram](image)

- Contact on the flange
- Contact on the rolling tread

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**Difficulties of the geometric problem**

- Double contact
- Discontinuities and contact jumps (several contact points)

![Graph](image)
Normal contact force

Options for determination of normal contact force

- **contact=geometric constraint**: $N$=Lagrange multiplier $\rightarrow$ contact zone by Hertz theory
  - Advantages: ability to tabulate the contact, numerically more stable
  - Drawbacks: not adapted to several contact points, contact area always assumed elliptical

- **contact=force element** with wheel-rail penetration $\rightarrow$ contact area and normal force by elastic forces.
  - Advantages: naturally adapted to several contact points
  - Drawbacks: computational burden, stiff motion equations

Tangential contact forces

Elastic deformation of wheel and rail

- Creepages (slips)
- Normal force
- Contact geometry

Kalker theory
Coulomb friction
Elasticity

Assessment of performances

**Major concerns**

- Stability, critical speed
- Ride quality – comfort
- UIC 518 criteria (tests): stability, track loads, derailment

**Main design variables**

- Global layout (distribution of bogies)
- Primary suspensions and contact geometry (stability and track loads)
- Secondary suspension (comfort)
Stability – kinematic yaw

The wheelset naturally follows the rail if:
- \( V_x, r_p \neq r_d \)
- \( V_y = V_d \)
- \( y = y \)

but oscillates with a wavelength:

\[
L = \frac{V}{f} = 2\pi \sqrt{\frac{r_i}{\delta}}
\]

(Klingel formula (no slip))

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Linear analyses

Equations of motion can be linearized about a stationary motion -> study of small perturbations

- Vehicle (bodies, points...)
- Velocity
- Equivalent excitation
- Track irregularities (PSD)

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Root locus

Root locus: evolution of poles (roots) with velocity

- Pole \( p = \sigma + j\omega \)
- Frequency \( f = \frac{\omega}{2\pi} \)
- Damping coefficient \( \xi \)

\[
\xi = \frac{\sigma}{\sqrt{\sigma^2 + \omega^2}}
\]

Stable (Re<0)

Unstable (Re>0)

10% limit
Root locus of a bogie

There exists a critical speed!

Critical speed vs concity

Tramway of Minneapolis

Independent wheels  
Rigid wheelsets
Tramway of Minneapolis

GraSMech – Multibody  91

Pole leading to instability at low conicity

Tramway of Minneapolis

GraSMech – Multibody  92

Pole leading to instability at high conicity

Nonlinear stability analysis

Increasing conicity

GraSMech – Multibody  93
Cityrunner

- Short car bodies: adapted to tortuous networks
- More car bodies than bogies
- Central car body not connected to the rail
- Comfort of the central car body driven by the global dynamics of the vehicle

Derailment study

For each rolling element (Nadal formula)

\[
\frac{Y}{Q} \leq \frac{\tan \alpha - f}{1 + f \tan \alpha}
\]

with

- \(Y\) total contact lateral force on the rolling element
- \(Q\) total contact vertical force on the rolling element
- \(\alpha\) angle of the cone described by the flange (\(\equiv 70\) deg)
- \(f\) friction coefficient (\(\equiv 0.3\))

Minneapolis

\[
\frac{Y}{Q} \leq 1.6
\]
Conclusions

- Reliable commercial softwares exist with many possibilities
- They are classically used in industry
- Attention to modelling mistakes!
- A prototype will always be built for final validation

Today Challenges

- Multi – physics modeling and simulation
- Real time computation
  optimization, «hardware in the loop» simulations, …
- Parameter identification
- Education
- …
Perspectives: multiphysics

- Mechatronics = science of motion control
  - Mechanism
  - Actuators
  - Sensors
  - Control law

Cars (suspension, ESP...)
High-speed machines
Large manipulators
Magnetic beams

Perspectives: multiphysics

- Integrated analysis and design
  - Multibody dynamics
  - Electronics
  - Hydraulics
  - Pneumatics
  - Electrical circuits
  - Piezoelectricity
  - Magnetics
  - ...

- Next step: micro-mechatronics
  - new technologies / new physical effects

Radial comb motor
Micropump
Bistable mechanism

Multibody System Optimization

- Optimization of « dynamical » performances
  - Fast computation of the cost function (ex. symbolic model)
  - Optimization algorithms: deterministic or stochastic methods

- Geometrical/morphological optimization of mechanisms
  - Robust assembly method (loop closure)
  - Problem of local optima (related to mechanism reconfiguration)
  - Problem of computer time (=> parallel computation)

- Illustration
  - Car suspension
  - Planar mechanism
  - Acherman steering mechanism
### Multibody System Optimization

**Student final project**

**Suspension geometrical optimization**

*for lateral dynamic stability*

**Test:** lane change at 65 km/h

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**Multibody System Optimization**

**Optimal design of path-following mechanisms**

- Deformation of mechanism to circumvent assembling constraints

- **objective:** strain energy of the bars

- **parameters:** natural lengths of the springs

- Use of natural coordinates

- **Non-Linear Least Squares Optimization**

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**Multibody System Optimization**

- Different starting points → different local optima

- **Example:** Six-bar steering linkage mechanism to fulfill the Ackermann condition

- Other design criteria to choose best mechanism (e.g., robust design)
Multibody Real time computation

For “greedy” simulations, such as:
- Sensitivity analysis
- Identification process (many run of the model / many “exciting” trajectories
- Optimization of a dynamic transient behaviour

For real “Real time” requirement
- Real time control of robots (including an inverse dynamic model)
- “Hardware in the loop” analysis
- Others (Haptic systems, games, …)

Example: semi-active control of car

Non-Model-based controller \(\rightarrow\) Model-based controller

Multibody Education via projects!

At the end of a MBS project, students should be able to:
- formulate consistent hypotheses (Engineering)
- manipulate 3D kinematic tools (Physics)
- use the Newton-Raphson method to find system equilibrium (Numerical methods)
- build a well-structured simulation program (Programming)
- make a 3D drawing of the vehicle and a 3D virtual animation
- analyse their results (Physics)
- question their model (Engineering)

at the basis of the final examination
Multibody Education via projects!

Given...
...a real system
...numerical methods
...a group
...a CAD system
...programming tools
...theoretical formalisms
...system animation
...system analysis
...system parameterization

Problem to solve: truck “jacknifing”

Typical Results

Solved!
Problem to solve: Sidecar side-skidding

Solved?