

Université de Mons  
Faculté Polytechnique – Service de Mécanique Rationnelle, Dynamique et Vibrations  
31, Bld Dolez - B-7000 MONS (Belgique)  
065/37 42 15 – georges.kouroussis@umons.ac.be

---



ASSOCIATION  
FRANÇAISE  
DE MECANIQUE

GST Vibrations et Bruit

Appel à communication

3<sup>ème</sup> Colloque  
« Analyse Vibratoire  
Expérimentale »

20 et 21 Novembre 2012



<http://www.enivl.fr>  
Rubrique Colloque AVE2012

G. Kouroussis, L. Ben Fekih, C. Conti, O. Verlinden, EasyMod : du développement d'un toolbox sous MatLab vers l'enseignement des bases de l'analyse modale expérimentale, 3<sup>ème</sup> Colloque « Analyse vibratoire Expérimentale », Blois (France), 20 et 21 novembre 2012.

# EASYMOD : DU DÉVELOPPEMENT D'UN TOOLBOX SOUS MATLAB VERS L'ENSEIGNEMENT DES BASES DE L'ANALYSE MODALE EXPÉRIMENTALE

## *EasyMod: from the development of a MatLab toolbox to the teaching of experimental modal analysis fundamentals*

Georges Kouroussis<sup>1</sup>, Lassaad Ben Fekih<sup>1</sup>, Calogero Conti<sup>1</sup> et Olivier Verlinden<sup>1</sup>

<sup>1</sup> Département de Mécanique Rationnelle, Dynamique et Vibrations, Université de Mons, Faculté Polytechnique, B-7000 MONS, BELGIQUE, e-mail : [georges.kouroussis@umons.ac.be](mailto:georges.kouroussis@umons.ac.be), [lassaad.benfekih@umons.ac.be](mailto:lassaad.benfekih@umons.ac.be), [calogero.conti@umons.ac.be](mailto:calogero.conti@umons.ac.be), [olivier.verlinden@umons.ac.be](mailto:olivier.verlinden@umons.ac.be)

**Mots clés :** Analyse modale, formation en vibrations, mesures vibratoires, logiciel matriciel, méthodes SISO/SIMO/MIMO.

**Keywords:** *Modal analysis, vibrations education, vibration measurement, matrix manipulation software, SISO/SIMO/MIMO methods.*

## Résumé

*Ce papier présente le développement de l'outil EasyMod pour l'analyse modale expérimentale. Outre les méthodes d'identification de paramètres modaux, il intègre une série de fonctionnalités permettant une analyse complète, avec une validation sur base, par exemple, du modal assurance criterion ou d'une animation graphique grâce au logiciel EasyAnim, développé par ailleurs dans ces mêmes objectifs pédagogiques. L'outil est proposé sous couvert de la licence GNU, offrant ainsi au monde scientifique la possibilité de l'utiliser librement mais également de le modifier au travers de la réutilisation des codes sources. Le lien avec les logiciels commerciaux est également possible, grâce à l'utilisation de fichiers standardisés qui restent compatibles avec la plupart des logiciels. Des exemples, tirés de nos enseignements, illustrent l'approche didactique proposée, en utilisant des données numériques (établies à partir des matrices de masse, de raideur et d'amortissement issues d'un modèle analytique) ou des données expérimentales (raquette de tennis, cadre de vélo, ski alpin, ...).*

## 1 Introduction

Experimental modal analysis (EMA) is a fundamental technique which has been rapidly developed the last 30 years. It embraces a wide range of disciplines, and is considered, from the start, as a useful tool to study and illustrate the dynamic characteristics of a structure, since its scope is considerable [1]. It is now generally included in the process of structure design and modification, as illustrated in Figure 1. The term "experimental modal analysis" is retained in a large scale. Modal analysis is defined as the study of the dynamic characteristics of a mechanical structure. The theory not only emphasizes modal identification techniques, but also the associated signal processing and the frequency response function (FRF) testing.

From the last thirty years, many experimental modal analysis software packages have been developed [2]. Teaching modal analysis to engineering students responds to a growing industrial demand. At the undergraduate level, modal analysis courses treat the structural

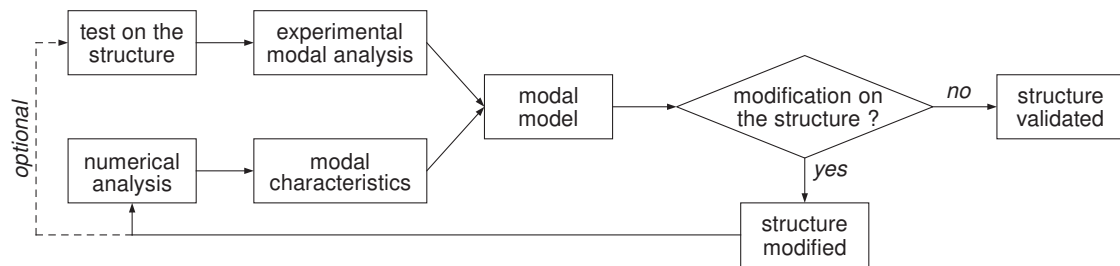


Fig. 1. Scheme of the structural dynamics modification process.

dynamics in the modal base, which presents the undeniable advantage to decouple motion equations, emphasizing the structure modal properties by using the mathematical basis of deriving the eigenfrequencies and the eigenvectors for time domain response data. Although the commercial tools provide a support for illustrating these mathematical concepts, they often mask a number of features which are important for a good understanding. It is therefore difficult to estimate the importance of measurement methods, the choice of a given frequency range or the selection of relevant coordinates. Initially composed by simple SDOF methods (peak picking/mode picking, circle-fit or line-fit), commercial software packages have been updated with new complex methods, allowing rapid and efficient analyses. Indeed, simple methods have been abandoned in favour of MDOF ones. Commercial tools are developed with the concerns of calculation power, user-friendliness and efficiency but represent a human and financial investment that the industry cannot afford if the application is not inside its core business. Nonetheless, these simple methods are very interesting for the education and allow a progressive approach in the modal analysis fundamentals teaching. Afterwards, the understanding of complex methods, like least-square complex exponential (LSCE) method or polymax estimator [3], which are considered as "industry-standard" time domain estimation methods, is facilitated, by pointing out their main advantages. A progressive approach is thus preferred, from simple to recent algorithms. The latter are essential for deriving better models of large and complex tested structures, only if the user well understands what he does.

To fill this gap, the *Department of Theoretical Mechanics, Dynamics and Vibrations* has developed some *MatLab* routines, which have been included in the so-called toolbox *EasyMod*, working under *MatLab*. The power of matrix manipulation added to a user-friendly platform are elements paving for this choice, in addition to the fact that *MatLab* is a reference program in engineering. *EasyMod* toolbox is presented as an alternative for studying EMA. This tool has the advantage to be open source, and can consequently be adapted by the engineering community according to educational purposes. This paper presents the adopted philosophy allowing an easy use through some interesting examples (analytical model, bike frame, squash racket and alpine ski). The toolbox structure is revealed and the illustration of theoretical concepts is shown.

## 2 Presentation of the tool EasyMod

### 2.1 Functioning and functionalities

The functioning and the possibilities of *EasyMod* are illustrated in Figure 2 with the emphasis on the identification and validation methods. To date, three identification methods have been implemented:

- two SISO (single-input/single-output) methods: the *circle-fit* and *line-fit*;
- one MIMO (multi-input/multi-output) method<sup>1</sup>: the *LSCE* (least-square complex exponential).

<sup>1</sup>The LSCE method is exactly a SIMO method (or a MISO method, according to the Betti-Maxwell reciprocity theorem). Data relative to several references can be used. However the model obtained by this method does not take specific advantage of this.

Several MatLab/Scilab functions have been developed for various applications in structural dynamics:

- reading and writing of UFF (universal file format) files,
- generating of frequency response functions from mass  $M$ , stiffness  $K$ , damping  $C$  matrices defining discrete systems,
- mode indicators (sum of FRFs, sum of FRFs real part and sum of FRFs imaginary part) and their visualization,
- modal assurance criterion and modal collinearity for a comparison of two sets of analysis.

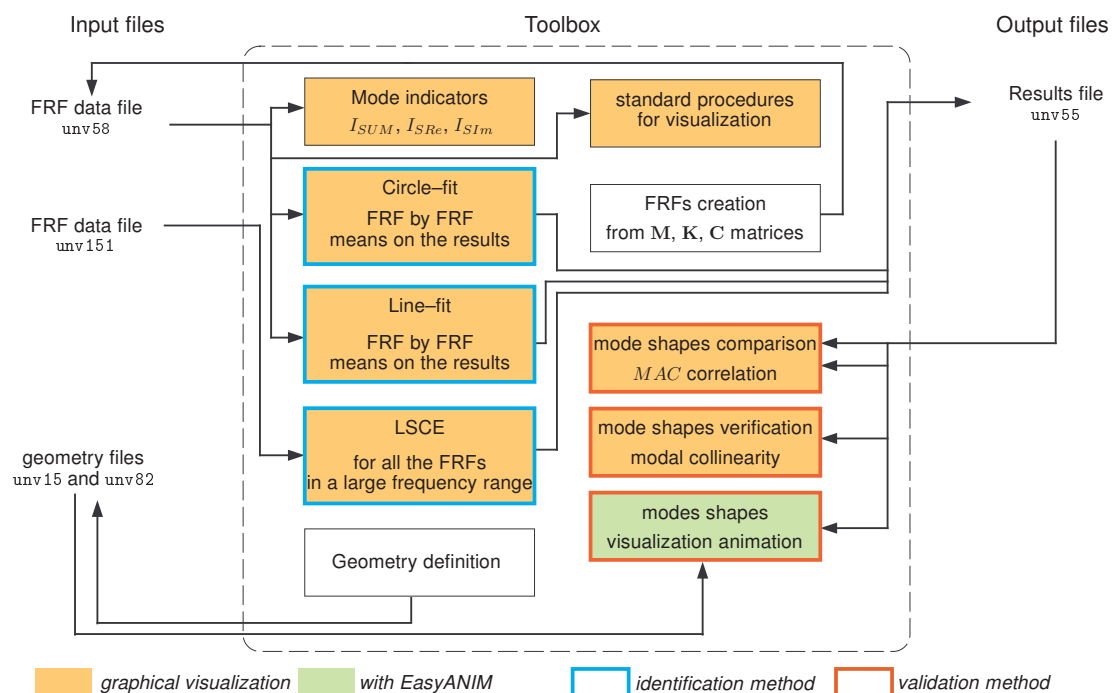


Fig. 2. Schematic operating diagram of toolbox EasyMod.

By using the matrix abilities offered by *MatLab*, operations on frequency response function are well made easier. Among others, the generation of FRF from mass, damping and stiffness matrices is summarized into one function only. Notice that other *MatLab*-based solutions exist, as for example *SDTools* [4], but above all dedicated to commercial EMA and dynamic simulation for vibration problems.

All the information is saved in files so-called universal files, for whom the format is standard in the field of vibration/dynamic experimentation. The universal file format (UFF) is presented as an interesting way to initialize a project. UFF is the industry standard format for storage of geometry, DOF information and measurements, supporting both ASCII and binary format. Although some restrictions are still pointed out and the format is considered obsolete by the original developer, it presents a certain degree of universality [5] and remains a standard for the experimental dynamics community, especially in the area of modal analysis. In ASCII format, all file types present the same structure. A specific number, at the script heading, is dedicated to the content: geometries, FRF data or results. For example, the format 58 is associated to the FRF measurement. The format defines a header that contains general information about the data contained in the file (function type, response direction, ...) and channel specific information (channel name, units, data type, ...).

## 2.2 Identification methods

Identification functions are proposed, not only with a growing degree of complexity, but also for SISO/SIMO/MIMO comparison. SISO methods allow to well emphasize the contribution of a single mode of a structure, supposing that one term in the series form of the frequency response function  $H_{ij}$  dominates the whole expression

$$H_{ij} = \frac{X_i}{F_j} = \sum_{k=1}^n \frac{B_{ijk}}{\omega_k^2 - \omega^2 + j2\xi_k\omega\omega_k} \quad (\text{eq. 1})$$

where  $\omega_k$  (natural circular frequency),  $\xi_k$  (damping ratio) and  $B_{ijk}$  (modal constant) are the modal parameters of mode  $k$ . These methods exploit interesting properties of a FRF like the circularity in the immediate vicinity of resonance or the linearity of inverted FRFs, which are associated to important statement in modal analysis theory [6]. SIMO and MIMO represent a natural step towards industrial applications. The rapidity and precision (multi-output) as well as the distinction between two close modes in frequency (multi-input) are underlined through the use of an advanced method like LSCE. In this case, the use of indicators may turn out useful.

The matrix manipulation offered by *MatLab* presents the important advantage to easily analyse a FRF in any requested format through a user-friendly interface. For example, peak picking/mode picking method, which is certainly the oldest identification technique, does not require any specific procedure (curve-fitting, minimization,...) and modal parameters can be simply obtained from FRF's Bode diagram and imaginary amplitude in *MatLab* GUI.

Figure 3 displays an example of results obtained with *EasyMod*. The information provided by the circle-fit method gives interesting findings, by illustrating the circularity around the analysed natural frequency, and its dependency with the number of samples. The damping estimation by further examination of the spacing of the FRF points is also analysed, through the effect of the generalized version of the half-power points formula and its sensitivity with the selected points before or after resonance.

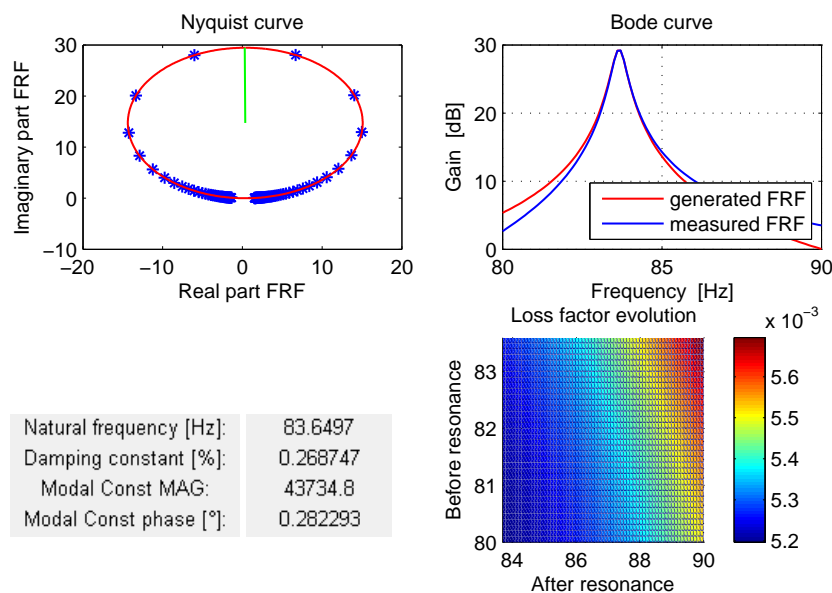


Fig. 3. Information provided by the circle-fit method.

The least-square complex exponential method allows a rapid estimation of modal frequency and damping values for several modes simultaneously. Since all the data is analyzed simultaneously, global analyses are obtained. It is based on the impulse response

functions, obtained from (eq. 1) by using the inverse Fourier transform:

$$h_{ij}(t) = \sum_{k=1}^{2n} r_{ijk} e^{\lambda_k t} \quad (\text{eq. 2})$$

where  $\lambda_k = -\xi_k \omega_k + j \omega_k \sqrt{1 - \xi_k^2}$  and  $r_{ijk}$  the residue of the receptance, taking into account that  $\lambda_{k+n} = \lambda_k^*$  and  $r_{ij(k+n)} = r_{ijk}^*$ . In addition to the stabilization diagram created to determine the optimal number of modes, a least-squares error chart and a conditioning error chart are provided by the corresponding function implemented under *EasyMod* (Figure 4).

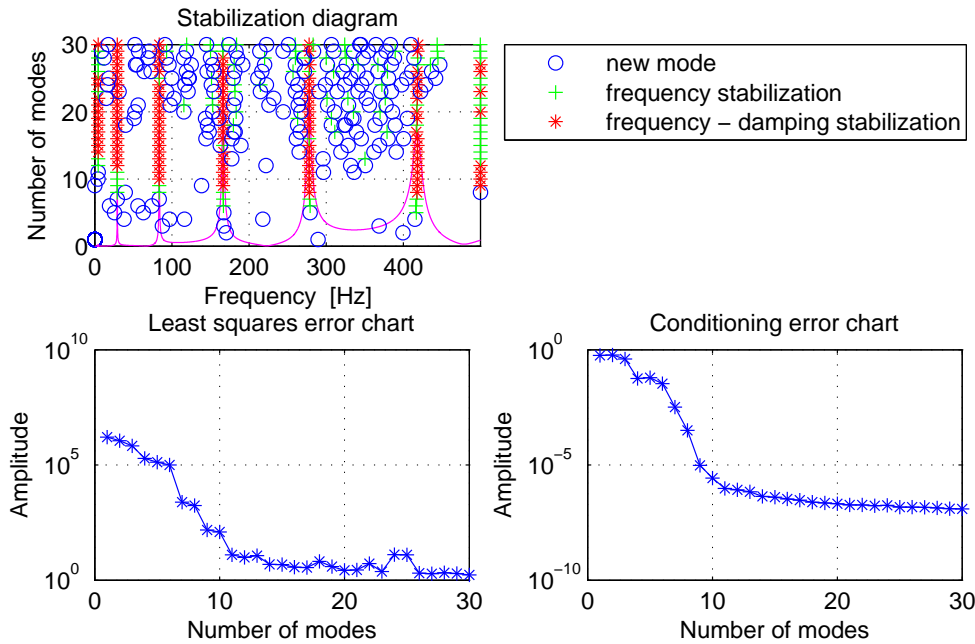


Fig. 4. Information provided by the LSCE method.

### 2.3 Validation methods

A good EMA software must provide several procedures that allow the modal model to be validated, once the modal parameters are determined. The most common validation procedure is to compare the FRFs synthesized from the modal model with the measured ones, which is easy to perform under *MatLab*. A widely used technique for comparing mode shapes is without any doubt the modal assurance criterion (MAC) which is defined by

$$MAC_{ij} = \frac{\left( \{\underline{\psi}_i\}^T \{\underline{\psi}_j\}^* \right)^2}{\|\underline{\psi}_i\|^2 \|\underline{\psi}_j\|^2} \quad (\text{eq. 3})$$

and gives a quantitative estimation of the modal vector consistency between two modal vectors  $\underline{\psi}_i$  and  $\underline{\psi}_j$ .

An indicator of the toolbox that defines whether a modal vector is essentially a normal mode is the modal phase collinearity (*MPC*), expressing the consistency of the linear relationship between the real and imaginary parts of each modal coefficient.

Another common method of modal model validation is to evaluate the modal vectors visually. For example, the software called *EasyAnim*, initially developed for teaching multi-body simulation [7], has been adapted to visualize mode shapes associated to analyses issued from *EasyMod* and/or from other commercial software supporting the UFF format (*Test.Lab*, *Cada-X*, *ME'scope*,...). *EasyAnim* is a home-made program to animate on the

screen scenes composed of nodes, edges and surfaces. It is based on OpenGL so that light and hidden-surfaces are treated automatically. The animation is built from two universal files, one for the geometry (consisting of a universal block 15 and a universal block 82), and the other for the modes (each mode in a universal block 55).

### 3 Studied examples

This section presents some interesting analytical and experimental examples treated with the help of *EasyMod*. They illustrate the possibilities of *EasyMod* to analyse any kind of mechanical structures. The choice of some of these structures is motivated by the student relevance for sport activities, which can find a real interest rather than simple structure like cantilever beams or plates.

#### 3.1 A 11-dof model

The analytical system is a noise-free 11-dof system [8] excited by three incoherent random sources at points 1, 2 and 9 and schematically illustrated on Figure 5. *EasyMod* provides functions for generating FRFs from matrices  $M$ ,  $C$  and  $K$  of the system and for saving it in UFF files. Therefore, it is possible to produce sets of data for the students as if they have been measured in the "virtual system", while defining variants by changing the damping level of the model. This example is typically well adapted to analyse the reliability of identification methods. Interested reader can find a detailed analysis of this system subsystems with the interest of *EasyMod* in [9].

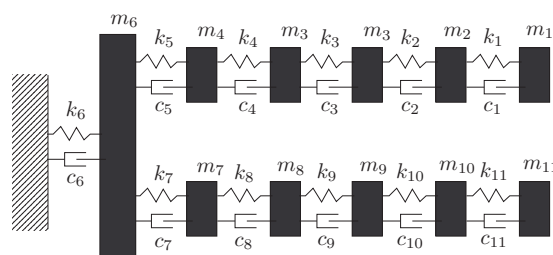


Fig. 5. 11-dof discrete model.

#### 3.2 A bike frame

The first experimental example is a bike frame (Figure 6). The FRF measurement is performed with a commercial software. This structure is interesting for various reasons:

- The analysis is limited to the out-of-plane modes of the frame in the frequency range [0 ; 800 Hz] with the emphasis on local and global modes shapes.
- It is a complete example for illustrating the concept of EMA, as well for the acquisition as for the modal identification.
- The structure boundary conditions is emphasized with the fixturing mechanism necessary to obtain the desired constraints (free condition).

#### 3.3 A squash racket

The tennis or squash racket is maybe the most popular sport equipment for which vibrations problem is related. The analysis is focused on the bending modes of a racket (Figure 7). Some student projects have been conducted to evaluate the possible influence of tennis vibration dampeners or the cause-effect chain with the tennis elbow (lateral epicondylitis) by measuring the transfer function between the racket and the forearm.



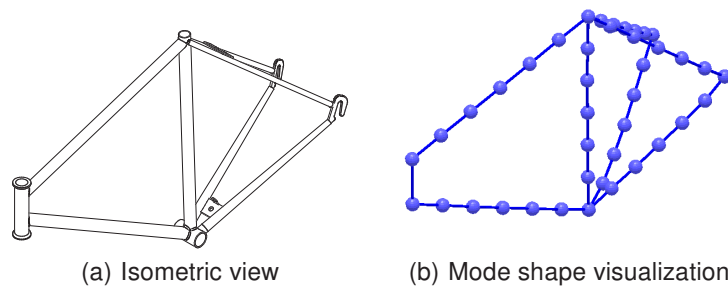


Fig. 6. Modal analysis of a bike frame.

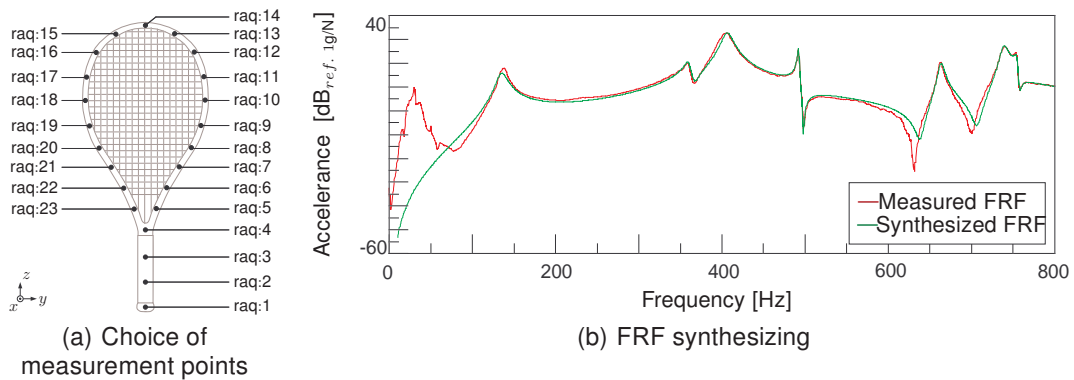


Fig. 7. Modal analysis of a squash racket.

### 3.4 An alpine ski

The alpine ski is a good example of a structure modal analysis taking into account the procedures motivated by state of the art specifications for vibration good practice. The ISO 6267:1980 standards [10] specifies the method for determining the natural frequency and half life related to the bending vibrations of this kind of equipment. The link with EMA results is performed by the students.

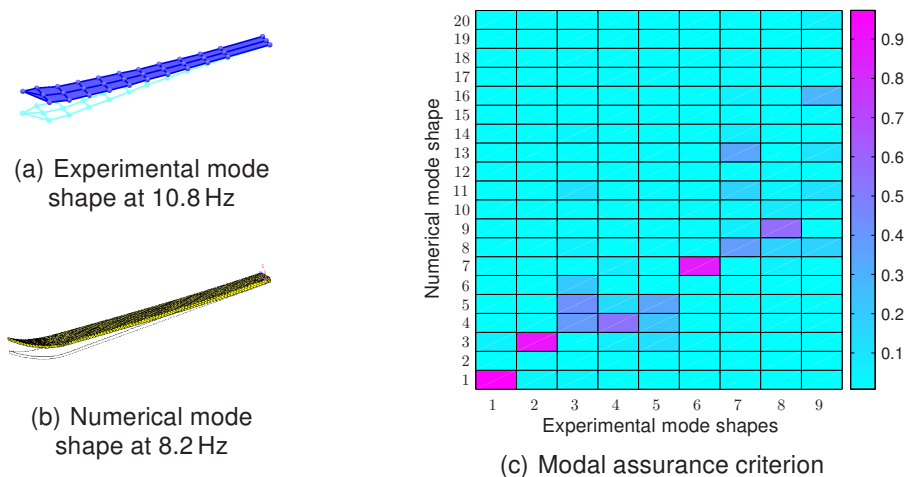


Fig. 8. Alpine ski vibrations: comparison between experimental and numerical results.

Figure 8 displays an example of correlation based on an alpine ski where measurement has been made according the ISO 6267:1980 standards (clamped binding mounting area)



with the emphasis on a finite element model updating. A good correlation is retained for some mode shapes by visualizing the *MAC* matrix (Figure 8(c)). The inconsistency of some modes can illustrate the possible cause (inaccurate finite element model, presence of noise or non-linearities in the measured data), initiating a motivated discussion with the students.

## 4 Conclusion

In this paper, a modal analysis toolbox has been presented for which students can visualize and illustrate the base fundamentals of the modal theory. All the presented examples show the possibilities of *EasyMod* to treat complex structures through various identification methods, simple SDOF and SISO methods (like circle-fit or line-fit) as well as MDOF and SIMO methods (LSCE). Since commercial software packages propose only one or two recent methods, *EasyMod* represents an interesting alternative to deal with the students in a course of modal analysis. As it is presented as an open-source framework, all the engineering community can develop their own contribution, in the scope of modal analysis teaching. The use of UFF files allows to make the link with commercial data acquisition software or modal parameter extraction programs, for a general-purpose use.

The toolbox can be downloaded in our web site where the interested reader can find additional information (<http://mecara.fpms.ac.be/EasyMod>).

## References

- [1] D. J. Ewins. "Modal analysis as a tool for studying structural vibration". In: S. G. Braun, editor, *Mechanical Signature Analysis*. Academic Press, London (UK), 1986. ISBN 978-0-12127-255-5.
- [2] N. M. M. Maia et al. *Theoretical and Experimental Modal Analysis*. John Wiley & Sons, Exeter (UK), 1997. ISBN 978-0-86380-208-9.
- [3] P. Guillaume, P. Verboven, S. Vanlanduit, H. Van der Auweraer, and B. Peeters. "A poly-reference implementation of the least-squares complex frequency-domain estimator". In: *IMAC 21, the International Modal Analysis Conference*, Kissimmee (USA), 3-6 February 2003.
- [4] SDTools. SDT Structural Dynamics Toolbox. <http://www.sdtools.com>, 2012.
- [5] A. W. Phillips and R. J. Allemang. "Development of a long term viable dynamic data archive format". In: *Modal Analysis Topics, Volume 3, Proceedings of the 29th IMAC, A Conference on Structural Dynamics*, Kissimmee (USA), 31 January - 3 February 2011. ISBN 978-1-44199-298-7.
- [6] D. J. Ewins. *Modal Testing: Theory and Practice*. John Wiley & Sons, Great Yarmouth (UK), 1991. ISBN 978-0-86380-218-8.
- [7] O. Verlinden, G. Kouroussis, and C. Conti. "EasyDyn: A framework based on free symbolic and numerical tools for teaching multibody systems". In: *ECCOMAS Thematic Conference Multibody Dynamics 2005*, Madrid (Spain), 21-24 June 2005.
- [8] N. Niedbal and E. Klusowski. "Optimal exciter placement and force vector tuning required for experimental modal analysis". In: *ESA SP-321 (Volume 1), Spacecraft Structures and Mechanical Testing, Proceedings of the International Conference Held at ESTEC*, Noordwijk (The Netherlands), 24-26 April 1991.
- [9] G. Kouroussis, L. Ben Fekih, C. Conti, and O. Verlinden. "EasyMod: A MatLab/SciLab toolbox for teaching modal analysis". In: *19th International Congress on Sound and Vibration (ICSV19)*, Vilnius (Lithuania), 08-12 July 2012. ISBN 978-6-09459-079-5.
- [10] International Organization for Standardization. *ISO 6267: Alpine skis — Measurement of bending vibrations*, 1980.