Parallel machine scheduling problem with single server

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Problem and objective:

<table>
<thead>
<tr>
<th>M3</th>
<th>M2</th>
<th>M1</th>
<th>Server</th>
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<tbody>
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<td><img src="setup_processing.png" alt="Diagram" /></td>
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Assumptions:

- Jobs are available at time zero.
- Preemption is not allowed.
- During the setup operation, both the machine and the server are occupied.
- The processing of each operation starts immediately after the end of setup.
- Setup operations are independent of the sequence and machines.

Some applications:

Network computing
Logistic
Production

Mathematical formulation:

\[ \min C_{\text{max}} \]
\[ \text{s.t.} \]
\[ C_{\text{max}} \geq C_i \]
\[ \sum_{k=1}^{m} x_{ik} = 1 \]
\[ C_i = S_{i+n} + p_{i+n} + p_i \]
\[ C_j \leq S_{j+n} + B(3 - x_{jk} - x_{jk} - z_{ij}) \]
\[ \forall (i,j) \in N^2, i < j, \forall k \in M \]
\[ C_j \leq S_{j+n} + B(2 - x_{jk} - x_{jk} + z_{ij}) \]
\[ \forall (i,j) \in N^2, i < j, \forall k \in M \]
\[ S_{i+n} + p_{i+n} \leq S_{j+n} + B(1 - z_{ij}, i+j) \]
\[ \forall (i,j) \in N^2, i < j \]
\[ S_{i+n} + p_{i+n} \leq S_{j+n} + B(z_{i+n}, j+n) \]
\[ x_{ik} \in \{0,1\} \]
\[ z_{ij} \in \{0,1\} \]
\[ S_{i+n} \geq 0 \]

Proposition 1:

\[ LB_1 = \frac{1}{m} \sum_{i=1}^{m} (p_i + s_i) \]

is a lower bound for the problem of P, S||C_{\text{max}}.

Main results:

- New mathematical formulations.
- Several lower/upper bounds + mathematical formulations
- Valid inequalities + approximate solutions
- IMPROVED / EFFICIENT / OPTIMAL SOLUTIONS

References: